

MATHML

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Introduction

It is a well known fact that \TeX can do a pretty good job on typesetting math. This is one reason why many scientific articles, papers and books are typeset using \TeX . However, in these days of triumphing angle brackets, coding in \TeX looks more and more out of place.

From the point of view of an author, coding in \TeX is quite natural, given that some time is spent on reading the manuals. This is (I'm told) because not only the natural flow of the definition suits the way mathematicians think, but also because the author has quite some control over the way his thoughts end up on paper. It will be no surprise that switching to a more restricted way of coding, which also demands more keystrokes, is not on forehand considered to be better.

There are however circumstances that one wants to share formulas (or formula-like specifications) between several applications, one of which is a typesetting engine. In that case, a bit more work now, later saves you some headaches due to keeping the different source documents in sync.

As soon as coding math in angle brackets is discussed, those in favour stress that coding can be eased by using appropriate editors. Here we encounter a dilemma. For optimal usage, one should code in terms of content, that is, the principles that are expressed in a formula. Editors are not that strong in this area, and if they would be, editing would be not that much different from traditionally editing formulas: just keying in ideas using code that at first sight looks obscure. A more graphical oriented editor can help authors to compose formulas, but the underlaying coding will mainly be in terms of placing glyphs and boxes, and as a result the code will hardly be usable in other applications.

So either we code in terms of concepts, which permits sharing code among applications, and poses strong limitations on the influence of authors on the visual appearance. Or we use an interactive editor to fine tune the appearance of a formula and take for granted that reuse will be minimal or suboptimal.

In the following chapters we will discuss the mathematical language MATHML in the perspective of typography. As a typesetting vehicle, we have used \CONTeXt . However, the principles introduced here and the examples that we provide are independent of \CONTeXt . For a more formal exploration we recommend the MATHML specification.

This document is dedicated to all those \CONTeXt users who like typesetting math. I'm sure that my father, who was a math teacher, would have liked proofreading this document. His absence was compensated by Tobias Burnus, Wang Lei, Ton Otten, and members of the \CONTeXt mailing list who carefully read the text, corrected the errors in my math, tested the functionality, and made suggestions. Any remaining errors are mine.

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Hasselt, Januari 2001

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What is MATHML

<> *Backgrounds* MATHML showed up in the evolving vacuum between structural SGML markup and presentational HTML. Both SGML and HTML can be recognized by angle brackets. The disadvantage of SGML was that it was so open ended, that general tools could hardly be developed. HTML on the other hand was easy to use and became extremely popular and users as well as software vendors quickly spoiled the original ideas and created a mess. SGML never became really popular, but thanks to HTML people became accustomed to that kind of notation. So, when XML came around as a more restricted cousin of SGML, the world was kind of ready for it. It cannot be denied that by some clever marketing many of today's users think that they use something new and modern, while we are actually dealing with something from the early days of computing. A main benefit of XML is that it brought the ideas behind SGML (and medium neutral coding in general) to the users and at the same time made a major cleanup of HTML possible.

About the same time, MATHML was defined, both to bring math to the www, and to provide a way of coding math that will stimulate sharing the same code between different applications. At the end of 2000, the MATHML version 2 draft became a recommendation.

Now, imagine that we want to present a document on the internet using a format like HTML, either for viewing or for aural reproduction. Converting text and graphics is, given proper source coding, seldom a problem, but converting formulas into some angle bracket representation is more tricky. A way out of this is MATHML's presentational markup.

$$a + b = c$$

This simple formula, when coded in T_EX, looks like:

\$\$ a + b = c \$\$

In presentational MATHML we get:

```
<math>
  <mrow>
    <mi> a </mi>
    <mo> + </mo>
    <mi> b </mi>
    <mo> = </mo>
    <mi> c </mi>
  </mrow>
</math>
```

In presentational MATHML, we use mostly begintags (`<mi>`) and end tags (`</mi>`). The `row` element is the basic building block of a formula. The `mi` element specifies a math identifier and `mo` is used for operators. In the process of typesetting, both are subjected to interpretation in order to get the best visualization.

Converting \TeX code directly or indirectly, using the DVI output or even in-memory produced math lists, has been one of the driving forces behind presentational MATHML and other math related DTD's like EUROMATH. One may wonder if there are sound and valid reasons for going the opposite way. You can imagine that a converter from \TeX to MATHML produces `menclose`, `mSPACE`, `mstyle` and other elements that can have many spacing related attributes, but I wonder if any author is willing to think in those quantities. Visual editors of course are good candidates for producing presentational MATHML.

But wouldn't it be more efficient if we could express ideas and concepts in such a way that they could be handled by a broad range of applications, including a typesetting engine? This is why, in addition to presentational MATHML, there is also content MATHML. The previous formula, when coded in such a way, looks like:

```
<math>
  <apply> <eq/>
    <apply> <plus/>
      <ci> a </ci>
      <ci> b </ci>
    </apply>
  <ci> c </ci>
</apply>
</math>
```

This way of defining a formula resembles the so called polish (or stackwise) notation. Opposite to presentational markup, here a typesetting engine has to find out in what order and what way the content has to be presented. This may seem a disadvantage, but in practice implementing content markup is not that complicated. The big advantage is that, once we know how to typeset a concept, \TeX can do a good job, while in presentational markup much hard coded spacing can spoil everything. One can of course ignore specific elements, but it is more safe to start from less and enhance, than to leave away something with unknown quantities.

Instead of using hard coded operators as in presentational MATHML, content markup uses empty elements like `<plus/>`. Many operators and functions are predefined but one can also define his own, which is not entirely en par with the concept.

Of course the main question to be answered now is to what extent the author can influence the appearance of a formula defined in content markup. Content markup has the

advantage that the results can be more consistent, but taking away all control is counterproductive. The MATHML level 2 draft mentions that this level covers most of the pre university math. If so, that is a proper starting point, but especially educational math often has to be typeset in such ways that it serves its purpose. Also, (re)using the formulas in other applications (simulators and alike) is useful in an educational setting, so content markup is quite suitable.

How do we combine the advantages of content markup with the wish of an author to control the visual output and at the same time get an as high as possible typeset result. There are several ways to accomplish this. One is to include in the document source both the content markup and the $\text{T}_\text{E}\text{X}$ specific code.

```
<math>
  <semantics>
    <apply> <eq/>
      <apply> <plus/>
        <ci> a </ci>
        <ci> b </ci>
      </apply>
    </apply>
    <ci> c </ci>
    <annotation encoding="TeX">a+b=c</annotation>
  </semantics>
</math>
```

The *annotation* element is one of the few that is permitted inside the *math* element. In this example, we embed pure $\text{T}_\text{E}\text{X}$ code, which, when enabled is typeset in math mode. It will be clear that for a simple formula like this one, such redundant coding is not needed, but one can imagine more complicated formulas. Because we want to limit the amount of work, we prefer just content markup.

<> *Two methods* The best way to learn MATHML is to key in formulas, so that is what we did as soon as we started adding MATHML support to $\text{CON}\text{T}_\text{E}\text{X}\text{T}$. In some areas, MATHML provides much detail (many functions are represented by elements) while in other areas one has to fall back on the more generic function element or a full description. Compare the following definitions:

```
<math> <apply> <sin/> <ci> x </ci> </apply> </math>
<math> <mrow> <mo> sin </mo> <mi> x </mi> </mrow> </math>
```

We prefer the first definition because it is more structured and gives more control over the result. There is only one ‘unknown’ quantity, x , and from the encapsulating element *ci* we know that it is an identifier.

$$\sin x$$

$$\sin x$$

In the content example, from the *apply sin* we can deduce that the following argument is an operand, either an *apply*, or an *ci* or *cn*. In the presentational alternative, the following elements can be braces, a math identifier, a row, a sequence of identifiers and operators, etc. There, the look and feel is hard coded.

`<?context-mathml-directive function reduction no ?>`

This directive, either issued in the XML file, or set in the style file, changes the appearance of the function, but only in content markup. It is because of this feature, that we favour content markup.

$$\sin(x)$$

$$\sin x$$

Does this mean that we can cover everything with content markup? The answer to this is still unclear. Consider the following definition.

$$\int \frac{1}{\cos(ax) 1 \pm \sin(ax)} dx = \mp \frac{1}{2a 1 \pm \sin(ax)} + \frac{1}{2a} \log \tan \left(\frac{\pi}{4} + \frac{ax}{2} \right)$$

Here we combine several cases in one formula by using \pm and \mp symbols. Because we only have *plus* and *minus* elements, we have to revert to the generic function element *fn*. We show the complete definition of this formula.

```
<math>
  <apply> <eq/>
    <apply> <int/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <times/>
          <apply> <cos/>
            <apply> <times/>
```

```

        <ci> a </ci>
        <ci> x </ci>
    </apply>
</apply>
<apply> <fn> <ci> &plusminus; </ci> </fn>
    <cn> 1 </cn>
    <apply> <sin/>
        <apply> <times/>
            <ci> a </ci>
            <ci> x </ci>
        </apply>
    </apply>
</apply>
</apply>
</apply>
<apply> <plus/>
    <apply> <fn> <ci> &minusplus; </ci> </fn>
        <apply> <divide/>
            <cn> 1 </cn>
            <apply> <times/>
                <cn> 2 </cn>
                <ci> a </ci>
            <apply> <fn> <ci> &plusminus; </ci> </fn>
                <cn> 1 </cn>
                <apply> <sin/>
                    <apply> <times/>
                        <ci> a </ci>
                        <ci> x </ci>
                    </apply>
                </apply>
            </apply>
        </apply>
    </apply>
</apply>
</apply>
<apply> <times/>
    <apply> <divide/>
        <cn> 1 </cn>

```

```

    <apply> <times/>
      <cn> 2 </cn>
      <ci> a </ci>
    </apply>
  </apply>
  <apply> <log/>
    <apply> <tan/>
      <apply> <plus/>
        <apply> <divide/>
          <ci> &pi; </ci>
          <cn> 4 </cn>
        </apply>
        <apply> <divide/>
          <apply> <times/>
            <ci> a </ci>
            <ci> x </ci>
          </apply>
          <cn> 2 </cn>
        </apply>
      </apply>
    </apply>
  </apply>
</math>

```

The MATHML parser and typesetting engine have to know how to handle these special cases, because the visualization depends on the function (or operator). Here both composed signs are treated like the plus and minus signs, but in other cases an embraced argument may be needed. Each special case needs a specific handler.

Presentation markup

If a document contains presentational MATHML, there is a good chance that the code is output by an editor. Here we will discuss the presentation elements that make sense for users when they want to manually code presentational MATHML. In this chapter we show the default rendering, later we will discuss options.

Although much is permitted, we advise to keep the code as simple as possible, because then $\text{T}_{\text{E}}\text{X}$ can do a rather good job on interpreting and typesetting it. Just let $\text{T}_{\text{E}}\text{X}$ take care of the spacing.

<> *mi*, *mn*, *mo* Presentational markup comes down to pasting boxes together in math specific ways. The basic building blocks are these three character elements.

$$x = 5$$

```
<math>
  <mrow>
    <mi> x </mi> <mo> = </mo> <mn> 5 </mn>
  </mrow>
</math>
```

<i>mi</i>	identifier	normally typeset in an italic font
<i>mn</i>	number	normally typeset in a normal font
<i>mo</i>	operator	surrounded by specific spacing

<> *mrow* The previous example demonstrated the use of *mrow*, the element that is used to communicate the larger building blocks. Although this element from the perspective of typesetting is not always needed, by using it, the structure of the formula in the document source is more clear.

<> *msup*, *msub*, *msubsup* Where in content markup super and subscript are absent and derived from the context, in presentational markup they are quite present.

$$x_1^2$$

```
<math>
  <msup>
    <msub> <mi> x </mi> <mn> 1 </mn> </msub>
```

```

    <mn> 2 </mn>
  </msup>
</math>

```

$$x_1^2$$

```

<math>
  <msubsup>
    <mi> x </mi>
    <mn> 1 </mn>
    <mn> 2 </mn>
  </msubsup>
</math>

```

Watch the difference between both definitions and appearances. You can influence the default behaviour with processing instructions.

<> *mfrac* Addition, subtraction and multiplication is hard coded using the *mo* element with +, −, and × (or nothing). You can use / for division, but for more complicated formulas you have to fall back on fraction building. This is why MATHML provides the *mfrac*.

$$\frac{x + 1}{y + 1}$$

```

<math>
  <mfrac>
    <mrow> <mi> x </mi> <mo> + </mo> <mn> 1 </mn> </mrow>
    <mrow> <mi> y </mi> <mo> + </mo> <mn> 1 </mn> </mrow>
  </mfrac>
</math>

```

You can change the width of the rule, but this is generally a bad idea. For special purposes you can set the line thickness to zero.

$$x \geq 2$$

$$y \leq 4$$

```

<math>
  <mfrac linethickness="0">
    <mrow> <mi> x </mi> <mo> &geq; </mo> <mn> 2 </mn> </mrow>
    <mrow> <mi> y </mi> <mo> &leq; </mo> <mn> 4 </mn> </mrow>
  </mfrac>
</math>

```

```
</mfrac>
</math>
```

<> *mfenced* Braces are used to visually group sub-expressions. In presentational MATHML you can either hard code braces, or use the *mfenced* element to generate delimiters automatically.

```
<mo>(</mo> <mi> x </mi> <mo> + </mo> <mn> 1 </mn> <mo>)</mo>
<mfenced> <mi> x </mi> <mo> + </mo> <mn> 1 </mn> </mfenced>
```

In CONTEXT , as much as possible, the operators and identifiers are interpreted, and when recognized treated according to their nature.

$$\frac{(x+1)(x-1)}{(y+1)(y-1)}$$

```
<math>
  <mfrac>
    <mrow>
      <mfenced> <mi> x </mi> <mo> + </mo> <mn> 1 </mn> </mfenced>
      <mfenced> <mi> x </mi> <mo> - </mo> <mn> 1 </mn> </mfenced>
    </mrow>
    <mrow>
      <mfenced> <mi> y </mi> <mo> + </mo> <mn> 1 </mn> </mfenced>
      <mfenced> <mi> y </mi> <mo> - </mo> <mn> 1 </mn> </mfenced>
    </mrow>
  </mfrac>
</math>
```

The braces adapt their size to the content. Their dimensions also depend on the way math fonts are defined. The standard $\text{T}_{\text{E}}\text{X}$ fonts will give the same height of braces around x and y , but in other fonts the y may invoke slightly larger ones.

$$(x,y,z)$$

```
<math>
  <mfenced open="(" close= ")" separators=",">
    <mi> x </mi> <mi> y </mi> <mi> z </mi>
  </mfenced>
</math>
```

The separators will adapt their size to the fenced content.

$$\left[\frac{1}{x} \mid \frac{1}{y} \mid \frac{1}{z} \right]$$

```
<math>
  <mfenced open="[" close="]" separators="|">
    <mfrac> <mn> 1 </mn> <mi> x </mi> </mfrac>
    <mfrac> <mn> 1 </mn> <mi> y </mi> </mfrac>
    <mfrac> <mn> 1 </mn> <mi> z </mi> </mfrac>
  </mfenced>
</math>
```

<> *msqrt*, *mroot* The shape and size of roots, integrals, sums and products can depend on the size of the content.

$$\sqrt{b}$$

```
<math>
  <msqrt>
    <mi> b </mi>
  </msqrt>
</math>
```

$$\sqrt[2]{b}$$

```
<math>
  <mroot>
    <mi> b </mi>
    <mn> 2 </mn>
  </mroot>
</math>
```

$$\sqrt[2]{\frac{1}{b}}$$

```
<math>
  <mroot>
    <mfrac> <mn> 1 </mn> <mi> b </mi> </mfrac>
    <mn> 2 </mn>
  </mroot>
</math>
```



```
</mroot>
</math>
```

$$\sqrt[3]{\frac{1}{a+b}}$$

```
<math>
  <mroot>
    <mfrac>
      <mn> 1 </mn>
      <mrow> <mi> a </mi> <mo> + </mo> <mi> b </mi> </mrow>
    </mfrac>
    <mn> 3 </mn>
  </mroot>
</math>
```

- <> *mtext* If you put text in a *mi* element, it will come out rather ugly. This is due to the fact that identifiers are (at least in T_EX) not subjected to the kerning that is normally used in text. Therefore, when you want to add some text to a formula, you should use the *mtext* element.

$$\frac{\textit{SomeText}}{\text{Some Text}}$$

```
<math>
  <mfrac>
    <mi> Some Text </mi>
    <mtext> Some Text </mtext>
  </mfrac>
</math>
```

- <> *mover, munder, munderover* Not all formulas are math and spacing and font rules may differ per discipline. The following formula reflects a chemical reaction.



```
<math>
  <mrow>
    <mrow>
```

```

    <mn> 2 </mn>
    <msub> <mtext> H </mtext> <mn> 2 </mn> </msub>
</mrow>
<mo> + </mo>
<msub> <mtext> 0 </mtext> <mn> 2 </mn> </msub>
<munder>
  <mo> &RightArrow; </mo>
  <mtext> explosion </mtext>
</munder>
<mrow>
  <mn> 2 </mn>
  <msub> <mtext> H </mtext> <mn> 2 </mn> </msub>
  <mtext> 0 </mtext>
</mrow>
</mrow>
</math>

```

The *munder*, *mover* and *munderover* elements can be used to put symbols and text or formulas on top of each other. When applicable, the symbols will stretch themselves to span the natural size of the text or formula.

<> *ms* This is a bit weird element. It behaves like *mtext* but puts quotes around the text.

$$\frac{\text{"Some Text"}}{\text{Some Text}}$$

```

<math>
  <mfrac>
    <ms> Some Text </ms>
    <mtext> Some Text </mtext>
  </mfrac>
</math>

```

You can specify the left and right boundary characters, either directly or (preferably) using entities like *quot*.

$$+A \text{ Famous Quotation} +$$

```

<math>
  <ms lquote="+" rquote="+"> A Famous Quotation </ms>
</math>

```

<> *menclose* This element is implemented but it is such a weird element that we don't yet describe it here.

<> *merror* There is not much chance that this element will end up in a math textbook, unless the typeset output of programs is part of the story.

$$\text{Are you kidding? } \frac{1+x}{0}$$

```
<math>
  <merror>
    <mtext> Are you kidding? &ThickSpace; </mtext>
    <mfrac>
      <mrow> <mn> 1 </mn> <mo> + </mo> <mi> x </mi> </mrow>
      <mn> 0 </mn>
    </mfrac>
  </merror>
</math>
```

<> *mmultiscripts*, *mprescripts* This element is one of the less obvious ones. The next two examples are taken from the specification. The *multiscripts* element takes an odd number of arguments. The second and successive child elements alternate between sub- and superscript. The empty element *none* —a dedicated element *mnone* would have been a better choice— serves as a placeholder.

$$R_i^j{}_{kl}$$

```
<math>
  <mmultiscripts>
    <mi> R </mi>
    <mi> i </mi>
    <none/>
    <none/>
    <mi> j </mi>
    <mi> k </mi>
    <none/>
    <mi> l </mi>
    <none/>
  </mmultiscripts>
</math>
```

```
</mmultiscripts>
</math>
```

The *mmultiscripts* element can also be used to attach prescripts to a symbol. The next example demonstrates this. The empty *prescripts* element signals the start of the prescripts section.

$$^{427}Qb_4$$

```
<math>
  <mmultiscripts>
    <mi> Qb </mi>
    <mn> 4 </mn>
    <none/>
    <mprescripts/>
    <mn> 427 </mn>
    <none/>
  </mmultiscripts>
</math>
```

- <> *mspace* Currently not all functionality of the *mspace* element is implemented. Over time we will see what support is needed and makes sense, especially since this command can spoil things. We only support the units that make sense, so units in terms of pixels—a rather persistent oversight in drafts—are kindly ignored.

use me with care

```
<math>
  <mrow>
    <mtext> use </mtext> <mspace width="1em" />
    <mtext> me </mtext> <mspace width="1ex" />
    <mtext> with </mtext> <mspace width="10pt"/>
    <mtext> care </mtext>
  </mrow>
</math>
```

- <> *mpantom* A phantom element hides its content but still takes its space. A phantom element can contain other elements.

who is afraid of elements

```
<math>
  <mrow>
    <mtext>    who is afraid of </mtext>    <mspace width=".5em" />
    <mphantom> phantom                </mphantom> <mspace width=".5em" />
    <mtext>    elements                </mtext>
  </mrow>
</math>
```

<> *mpadded* As with a few other elements, I first have to see some practical usage for this, before I implement the functionality needed.

<> *mtable, mtr, mtd, mlabeledtr* As soon as you want to represent a matrix or other more complicated composed constructs, you end up with spacing problems. This is when tables come into view. Because presentational elements have no deep knowledge about their content, tables made with presentational MATHML will in most cases look worse than those that result from content markup.

We have implemented tables on top of the normal XML (HTML) based table support in CONTEXt , also known as natural tables. Depending on the needs, support for the *mtable* element will be extended.

The *mtable* element takes a lot of attributes. When no attributes are given, we assume that a matrix is wanted, and typeset the content accordingly.

$$\begin{pmatrix} x_{1,1} & 1 & 0 \\ 0 & x_{2,2} & 1 \\ 0 & 1 & x_{3,3} \end{pmatrix}$$

```
<math>
  <mrow>
    <mo> ( </mo>
    <mtable>
      <mtr>
        <mtd> <msub> <mi> x </mi> <mn> 1,1 </mn> </msub> </mtd>
        <mtd> <mn> 1 </mn> </mtd>
        <mtd> <mn> 0 </mn> </mtd>
      </mtr>
    <mtr>
```

```

    <td> <mn> 0 </mn> </td>
    <td> <msub> <mi> x </mi> <mn> 2,2 </mn> </msub> </td>
    <td> <mn> 1 </mn> </td>
  </tr>
  <tr>
    <td> <mn> 0 </mn> </td>
    <td> <mn> 1 </mn> </td>
    <td> <msub> <mi> x </mi> <mn> 3,3 </mn> </msub> </td>
  </tr>
</mtable>
<mo> ) </mo>
</mrow>
</math>

```

100	100	100
10	10	10
1	1	1

```

<math>
  <mtable columnalign="left center right" color="red blue black">
    <mtr>
      <td frame="on"> <mn> 100 </mn> </td>
      <td> > <mn> 100 </mn> </td>
      <td> > <mn> 100 </mn> </td>
    </mtr>
    <mtr>
      <td> > <mn> 10 </mn> </td>
      <td frame="on"> <mn> 10 </mn> </td>
      <td> > <mn> 10 </mn> </td>
    </mtr>
    <mtr>
      <td> > <mn> 1 </mn> </td>
      <td> > <mn> 1 </mn> </td>
      <td frame="on"> <mn> 1 </mn> </td>
    </mtr>
  </mtable>
</math>

```

Although the underlying table mechanism can provide all the support needed (and even more), not all attributes are yet implemented. We will make a useful selection.

columnalign	keyword: left center (middle) right
columnspacing	a meaningful dimension
rowspacing	a meaningful dimension
frame	keyword: none (off) solid (on)
color	a named color identifier
background	a named color identifier

We only support properly named colors as back- and foreground colors. The normal \TeX color mapping mechanism can be used to remap colors. This permits (read: forces) a consistent usage of colors. If you use named backgrounds ... the sky is the limit.

- <> *malignmark* This element is used in tables and is not yet implemented, first because I still have to unravel its exact usage, but second, because it is about the ugliest piece of MATHML markup you will encounter.
- <> *mglyph* This element is for those who want to violate the ideas of general markup by popping in his or hers own glyphs. Of course one should use entities, even if they have to be defined.

▶ + ▶ = ▶▶

```
<math>
  <mrow>
    <mi> <mglyph fontfamily="navifont" index="2" alt="right"/>    </mi>
    <mo> + </mo>
    <mi> <mglyph fontfamily="navifont" index="2" alt="right"/>    </mi>
    <mo> = </mo>
    <mi> <mglyph fontfamily="navifont" index="6" alt="veryright"/></mi>
  </mrow>
</math>
```

- <> *mstyle* This element is implemented but not yet discussed since I want more control over its misuse.

<> *afterword* You may have noticed that we prefer content MATHML over presentational MATHML. So, unless you're already sick of any math coded in angle brackets, we invite you to read the next chapter too.

Content markup

In this chapter we will discuss the MATHML elements from the point of view of typesetting. We will not pay attention to other rendering techniques, like speech generation. Some elements take attributes and those often make more sense for other applications than for a typesetting engine like \TeX , which has a strong math engine that knows how to handle math.

- <> *apply* If you are dealing with rather ordinary math, you will only need a subset of content MATHML. For this reason we will start with the most common elements. When you key in XML directly, you will encounter the *apply* element quite often, even in a relatively short formula like the following.

```
<math> <apply> <minus/> <cn> 1 </cn> </apply> </math>
```

In most cases the *apply* element is followed by a specification disguised as an empty element.

- <> *ci, cn, sep* These elements are used to specify identifiers and numbers. Both elements can be made more explicit by using attributes.

type	set	use a representation appropriate for sets
	vector	mark this element as vector
	function	consider this element to be a function
	fn	idem

When *set* is specified, a blackboard symbol is used when available.

$$x \in \mathbb{N}$$

```
<math>
  <apply> <in/>
    <ci> x </ci>
    <ci type="set"> N </ci>
  </apply>
</math>
```

The *function* specification makes sense when the *ci* element is used in for instance a differential equation.

type	integer	a whole number with an optional base
	logical	a boolean constant
	rational	a real number
	complex-cartesian	a complex number in $x + iy$ notation
	complex	idem
	complex-polar	a complex number in polar notation ...

You're lucky when your document uses decimal notation, otherwise you will end up with long specs if you want to be clear in what numbers are used.

$$1A2C_{16} + 0101_{16} = 1B2D_{16}$$

```
<math>
  <apply> <eq/>
    <apply> <plus/>
      <cn type="integer" base="16"> 1A2C </cn>
      <cn type="integer" base="16"> 0101 </cn>
    </apply>
    <cn type="integer" base="16"> 1B2D </cn>
  </apply>
</math>
```

Complex numbers have two components. These are separated by the *sep* element. In the following example we see that instead of using a *ci* with set specifier, the empty element *complexes* can be used. We will see some more of those later.

$$2 + 5i \in \mathbb{C}$$

```
<math>
  <apply> <in/>
    <cn type="complex"> 2 <sep/> 5 </cn>
    <complexes/>
  </apply>
</math>
```

<> *eq, neq, gt, lt, geq, leq* Expressions, and especially those with *eq* are typical for math. Because such expressions can be quite large, there are provisions for proper alignment.

lt	$a < b$	leq	$a \leq b$
eq	$a = b$	neq	$a \neq b$
gt	$a > b$	geq	$a \geq b$

$$a \leq b \leq c$$

```
<math>
  <apply> <leq/>
    <ci> a </ci>
    <ci> b </ci>
    <ci> c </ci>
  </apply>
</math>
```

<> *equivalent, approx, implies* Equivalence, approximations, and implications are handled like *eq* and alike and have their own symbols.

$$a + b \equiv b + a$$

```
<math>
  <apply> <equivalent/>
    <apply> <plus/> <ci> a </ci> <ci> b </ci> </apply>
    <apply> <plus/> <ci> b </ci> <ci> a </ci> </apply>
  </apply>
</math>
```

This document is typeset with PDF_T_EX version 3.14159, and given that T_EX is written by a mathematician, it will be no surprise that:

$$3.14159 \approx \pi$$

```
<math>
  <apply> <approx/>
    <cn> 3.14159 </cn>
    <pi/>
  </apply>
</math>
```

$$x + 4 = 9 \Rightarrow x = 5$$

```

<math>
  <apply> <implies/>
    <apply> <eq/>
      <apply> <plus/>
        <ci> x </ci>
        <cn> 4 </cn>
      </apply>
      <cn> 9 </cn>
    </apply>
    <apply> <eq/>
      <ci> x </ci>
      <cn> 5 </cn>
    </apply>
  </apply>
</math>

```

<> *minus, plus* Addition and subtraction are main building blocks of math so you will meet them often.

$$37 - x$$

```

<math>
  <apply> <minus/>
    <cn> 37 </cn>
    <ci> x </ci>
  </apply>
</math>

```

In most cases there will be more than one argument to take care of, but especially *minus* will be used with one argument too. Although `<cn> -37 </cn>` is valid, using *minus* is sometimes more clear.

$$-37$$

```

<math>
  <apply> <minus/>
    <cn> 37 </cn>
  </apply>
</math>

```

You should pay attention to combinations of *plus* and *minus*. Opposite to presentational MATHML, in content markup you don't think and code sequential.

$$-x + 37$$

```
<math>
  <apply> <plus/>
    <apply> <minus/>
      <ci> x </ci>
    </apply>
  <cn> 37 </cn>
</apply>
</math>
```

- <> *times* Multiplication is another top ten element. Although 3p as content of the *ci* element would have rendered the next example as well, you really should split off the number and mark it as *cn*. When this is done consistently, we can comfortably change the font of numbers independent of the font used for displaying identifiers.

$$3p$$

```
<math>
  <apply> <times/>
    <cn> 3 </cn>
    <ci> p </ci>
  </apply>
</math>
```

In a following chapter we will see how we can add multiplication signs between variables and constants.

- <> *divide* When typeset, a division is characterized by a horizontal rule. Some elements, like the differential element *diff*, generate their own division.

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$$

This example also demonstrates how to mix *plus* and *minus*.

```

<math>
  <apply> <eq/>
    <apply> <plus/>
      <cn> 1 </cn>
      <apply> <minus/>
        <apply> <divide/>
          <cn> 1 </cn>
          <cn> 3 </cn>
        </apply>
      </apply>
    <apply> <divide/>
      <cn> 1 </cn>
      <cn> 5 </cn>
    </apply>
    <apply> <minus/>
      <apply> <divide/>
        <cn> 1 </cn>
        <cn> 7 </cn>
      </apply>
    </apply>
    <ci> &cdots; </ci>
  </apply>
  <apply> <divide/>
    <ci> &pi; </ci>
    <cn> 4 </cn>
  </apply>
</apply>
</math>

```

<> *power* In presentational MATHML you think in super- and subscripts, but in content MATHML these elements are not available. There you need to think in terms of *power*.

$$x^2 + \sin^2 x$$

```

<math>
  <apply> <plus/>
    <apply> <power/>
      <ci> x </ci>

```

```

    <cn> 2 </cn>
  </apply>
<apply> <power/>
  <apply> <sin/>
    <ci> x </ci>
  </apply>
  <cn> 2 </cn>
</apply>
</math>

```

The *power* element is clever enough to determine where the superscript should go. In the case of the sinus function, by default it will go after the function identifier.

<> *root, degree* If you study math related DTD's —this are the formal descriptions for SGML or XML element collections— you will notice that there are not that many elements that demand a special kind of typography: differential equations, limits, integrals and roots are the most distinctive ones.

$$\sqrt[3]{64} = 4$$

```

<math>
  <apply> <eq/>
    <apply> <root/>
      <degree> 3 </degree>
      <ci> 64 </ci>
    </apply>
    <cn> 4 </cn>
  </apply>
</math>

```

Contrary to *power*, the *root* element uses a specialized child element to denote the degree. The positive consequence of this is that there cannot be a misunderstanding about what role the child element plays, while in for instance *power* you need to know that the second child element denotes the degree.

<> *sin, cos, tan, cot, scs, sec, ...* All members of the family of goniometric functions are available as empty element. When needed, their argument is surrounded by braces. They all behave the same.

sin	arcsin	sinh	arcsinh
cos	arccos	cosh	arccosh
tan	arctan	tanh	arctanh
cot	arccot	coth	arccoth
csc	arccsc	csch	arccsch
sec	arcsec	sech	arcsech

These functions are normally typeset in a non italic (often roman) font shape.

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

By default the typesetting engine will minimize the number of braces that surrounds the argument of a function.

```
<math>
  <apply> <eq/>
    <apply> <sin/>
      <apply> <plus/>
        <ci> x </ci>
        <ci> y </ci>
      </apply>
    </apply>
    <apply> <plus/>
      <apply> <times/>
        <apply> <sin/>
          <ci> x </ci>
        </apply>
        <apply> <cos/>
          <ci> y </ci>
        </apply>
      </apply>
      <apply> <times/>
        <apply> <cos/>
          <ci> x </ci>
        </apply>
        <apply> <sin/>
          <ci> y </ci>
        </apply>
      </apply>
    </apply>
  </math>
```



```

    </apply>
  </apply>
</math>

```

You can specify π as an entity *pi* (coded as `π`) or as empty element *pi*. In many cases it is up to your taste which one you use. There are many symbols that are only available as entity, so in some respect there is no real reason to treat π different.

$$\cos \pi = -1$$

```

<math>
  <apply> <eq/>
    <apply> <cos/>
      <pi/>
    </apply>
    <apply> <minus/>
      <cn> 1 </cn>
    </apply>
  </apply>
</math>

```

<> *log, ln, exp* The *log* and *ln* are typeset similar to the previously discussed goniometric functions. The *exp* element is a special case of *power*. The constant *e* can be specified with *exponentiale*.

$$\ln(e + 2) \approx 1.55$$

```

<math>
  <apply> <approx/>
    <apply> <ln/>
      <apply> <plus/>
        <exponentiale/>
          <cn> 2 </cn>
        </apply>
      </apply>
    </apply>
    <cn> 1.55 </cn>
  </apply>
</math>

```

$$e^2 = 7.3890560989307$$

```
<math>
  <apply> <eq/>
    <apply> <exp/>
      <cn> 2 </cn>
    </apply>
    <cn> 7.3890560989307 </cn>
  </apply>
</math>
```

- <> *quotient, rem* The result of a division can be a rational number, so $\frac{5}{4}$ is equivalent to 1.25 and 1.25×4 gives 5. An integer division will give 1 with a remainder 2. Many computer languages provide a `div` and `mod` function, and since MATHML is also meant for computation, it provides similar concepts, represented by the elements *quotient* and *rem*. The representation of *quotient* is rather undefined, but the next one is among the recommended alternatives.

$$\lfloor a/b \rfloor$$

```
<math>
  <apply> <quotient/>
    <ci> a </ci>
    <ci> b </ci>
  </apply>
</math>
```

- <> *factorial* Showing the representation of a factorial is rather dull, so we will use a few more elements as well as a processing instruction to illustrate the usage of *factorial*.

$$n! = n \times (n - 1) \times (n - 2) \times \cdots \times 1$$

```
<math>
  <?context-mathml-directive times symbol yes ?>
  <apply> <eq/>
    <apply> <factorial/>
      <ci> n </ci>
    </apply>
  </math>
```

```

<apply> <times/>
  <ci> n </ci>
  <apply> <minus/> <ci> n </ci> <cn> 1 </cn> </apply>
  <apply> <minus/> <ci> n </ci> <cn> 2 </cn> </apply>
  <ci> &cdots; </ci>
  <cn> 1 </cn>
</apply>
</math>

```

The processing instruction is responsible for the placement of the \times symbols.

<> *min, max, gcd, lcm* These functions can handle more than two arguments. When typeset, these are separated by comma's.

$$z = \min \left\{ (x + y), 2x, \frac{1}{y} \right\}$$

```

<math>
  <apply> <eq/>
    <ci> z </ci>
    <apply> <min/>
      <apply> <plus/> <ci> x </ci> <ci> y </ci> </apply>
      <apply> <times/> <cn> 2 </cn> <ci> x </ci> </apply>
      <apply> <divide/> <cn> 1 </cn> <ci> y </ci> </apply>
    </apply>
  </math>

```

<> *and, or, xor, not* Logical expressions can be defined using these elements. The operations are represented by symbols and braces are applied when needed.

$$1001_2 \wedge 0101_2 = 0001_2$$

```

<math>
  <apply> <eq/>
    <apply> <and/>
      <cn type="integer" base="2"> 1001 </cn>
      <cn type="integer" base="2"> 0101 </cn>
    </apply>
  </math>

```

```

    </apply>
    <cn type="integer" base="2"> 0001 </cn>
  </apply>
</math>

```

<> *set*, *bvar* The appearance of a *set* depends on the presence of the child element *bvar*. In its simplest form, a set is represented as a list.

$$\{1,4,8\} \neq \emptyset$$

```

<math>
  <apply> <neq/>
    <set>
      <cn> 1 </cn>
      <cn> 4 </cn>
      <cn> 8 </cn>
    </set>
    <emptyset/>
  </apply>
</math>

```

A set can be distinguished from a vector by its curly braces. The simplest case is just a comma separated list. The next example demonstrates the declarative case. Without doubt, there will be other alternatives.

$$\{x \mid 2 < x < 8\}$$

```

<math>
  <set>
    <bvar><ci> x </ci></bvar>
    <condition>
      <apply> <lt/>
        <cn> 2 </cn>
        <ci> x </ci>
        <cn> 8 </cn>
      </apply>
    </condition>
  </set>
</math>

```

<> *list* This element is used in different contexts. When used as a top level element, a list is typeset as follows.

$$[1,1,3]$$

```
<math>
  <list>
    <cn> 1 </cn>
    <cn> 1 </cn>
    <cn> 3 </cn>
  </list>
</math>
```

When used in a context like *partialdiff*, the list specification becomes a subscript.

$$D_{1,1,3}f$$

```
<math>
  <apply> <partialdiff/>
    <list>
      <cn> 1 </cn>
      <cn> 1 </cn>
      <cn> 3 </cn>
    </list>
    <ci type="fn"> f </ci>
  </apply>
</math>
```

The function specification in this formula (which is taken from the specs) can also be specified as `<fn> <ci> f </ci> </fn>` (which is more clear).

<> *union, intersect, ...* There is a large number of set operators, each represented by a distinctive symbol.

union	$U \cup V$		
intersect	$U \cap V$		
in	$U \in V$	notin	$U \notin V$
subset	$U \subset V$	notsubset	$U \not\subset V$
prsubset	$U \subseteq V$	notprsubset	$U \not\subseteq V$
setdiff	$U \setminus V$		

These operators are applied as follows:

$$U \cup V$$

```
<math>
  <apply> <union/>
    <ci> U </ci>
    <ci> V </ci>
  </apply>
</math>
```

<> *conjugate, arg, real, imaginary* The visual representation of *conjugate* is a horizontal bar with a width matching the width of the expression.

$$\overline{x + iy}$$

```
<math>
  <apply> <conjugate/>
    <apply> <plus/>
      <ci> x </ci>
      <apply> <times/>
        <cn> &ImaginaryI; </cn>
        <ci> y </ci>
      </apply>
    </apply>
  </apply>
</math>
```

The *arg*, *real* and *imaginary* elements trigger the following appearance.

$$\arg(x + iy)$$

$$\Re(x + iy)$$

$$\Im(x + iy)$$

<> *abs, floor, ceiling* There are a couple of functions that turn numbers into positive or rounded ones. In computer languages names are used, but in math we use special boundary characters.

$$|-5| = 5$$

$$\lfloor 5.5 \rfloor = 5$$

$$\lceil 5.5 \rceil = 6$$

```

<math>
  <apply> <eq/>
    <apply> <abs/> <cn> -5 </cn> </apply>
    <cn> 5 </cn>
  </apply>
</math>
<math>
  <apply> <eq/>
    <apply> <floor/> <cn> 5.5 </cn> </apply>
    <cn> 5 </cn>
  </apply>
</math>
<math>
  <apply> <eq/>
    <apply> <ceiling/> <cn> 5.5 </cn> </apply>
    <cn> 6 </cn>
  </apply>
</math>

```

<> *interval* An interval is visualized as: $[1, 10]$. The *interval* element is a container element and has a begin and endtag. You can specify the closure as attribute:

$$(a, b]$$

```

<math>
  <interval closure="open-closed">
    <ci> a </ci>
    <ci> b </ci>
  </interval>
</math>

```

The following closures are supported:

open	(a, b)
closed	$[a, b]$
open-closed	$(a, b]$
closed-open	$[a, b)$

- <> *inverse* This operator is applied to a function. The following example demonstrates that this is one of the few cases (if not the only one) where the first element following an *apply* begintag is an *apply* itself.

$$\sin^{-1} x$$

```
<math>
  <apply>
    <apply> <inverse/> <sin/> </apply>
    <ci> x </ci>
  </apply>
</math>
```

- <> *reln* This element is a left-over from the first MATHML specification and its usage is no longer advocated. Its current functionality matches the functionality of *apply*.

- <> *cartesianproduct, vectorproduct, scalarproduct, outerproduct* Often the context of the formula will provide information of what kind of multiplication is meant, but using different symbols to represent the kind of product certainly helps.

$$a \times b$$

```
<math>
  <apply> <cartesianproduct/>
    <ci> a </ci>
    <ci> b </ci>
  </apply>
</math>
```

We have:

cartesian	$a \times b$
vector	$a \times b$

scalar $a \cdot b$
 outer $a \otimes b$

<> *sum, product, limit, lowlimit, uplimit, bvar* Sums, products and limits have a distinctive look, especially when they have upper and lower limits attached. Unfortunately there is no way to specify the x_i in content MATHML. In the next chapter we will see how we can handle that.

$$\sum_{i=1}^n \frac{1}{x}$$

```
<math>
  <apply> <sum/>
    <bvar> <ci> i </ci> </bvar>
    <lowlimit> <cn> 1 </cn> </lowlimit>
    <uplimit> <ci> n </ci> </uplimit>
    <apply> <divide/>
      <cn> 1 </cn>
      <ci> x </ci>
    </apply>
  </apply>
</math>
```

When we omit the limits, the *bvar* is still typeset.

$$\prod_i \frac{1}{x}$$

```
<math>
  <apply> <product/>
    <bvar>
      <ci> i </ci>
    </bvar>
    <apply> <divide/>
      <cn> 1 </cn>
      <ci> x </ci>
    </apply>
  </apply>
</math>
```

You can specify the condition under which the function is applied.

$$\prod_{x \in R} f(x)$$

```
<math>
  <apply> <product/>
    <bvar>
      <ci> x </ci>
    </bvar>
    <condition>
      <apply> <in/>
        <ci> x </ci>
        <ci type="set"> R </ci>
      </apply>
    </condition>
    <apply> <ci type="fn"> f </ci>
      <ci> x </ci>
    </apply>
  </apply>
</math>
```

$$\lim_{x \rightarrow 0} \sin x$$

```
<math>
  <apply> <limit/>
    <bvar>
      <ci> x </ci>
    </bvar>
    <lowlimit>
      <cn> 0 </cn>
    </lowlimit>
    <apply> <sin/>
      <ci> x </ci>
    </apply>
  </apply>
</math>
```

<> *int, diff, partialdiff, bvar, degree* These elements reach a high level of abstraction. The best way to learn how to use them is to carefully study some examples.

$$\frac{d \int_p^q f(x,a) dx}{da}$$

```

<math>
  <apply> <diff/>
    <bvar> <ci> a </ci> </bvar>
    <apply> <int/>
      <lowlimit> <ci> p </ci> </lowlimit>
      <uplimit> <ci> q </ci> </uplimit>
      <bvar> <ci> x </ci> </bvar>
      <apply>
        <fn> <ci> f </ci> </fn>
        <ci> x </ci>
        <ci> a </ci>
      </apply>
    </apply>
  </apply>
</math>

```

The *bvar* element is essential, since it is used to automatically generate some of the components that make up the visual appearance of the formula. If you look at the formal specification of these elements, you will notice that the appearance may depend on your definition. How the formula shows up, depends not only on the *bvar* element, but also on the optional *degree* element within.

$$f'$$

```

<math>
  <apply> <diff/>
    <ci> f </ci>
  </apply>
</math>

```

$$\frac{d^2 f(x)}{dx^2}$$

```

<math>
  <apply> <diff/>
    <bvar>
      <ci> x </ci>
    </bvar>
  </apply>
</math>

```

```

    <degree> <cn> 2 </cn> </degree>
  </bvar>
  <apply> <fn> <ci> f </ci> </fn>
    <ci> x </ci>
  </apply>
</math>

```

$$\frac{\partial^4 f}{\partial x^2 \partial y \partial x}$$

```

<math>
  <apply> <partialdiff/>
    <bvar>
      <degree> <cn> 2 </cn> </degree>
      <ci> x </ci>
    </bvar>
    <bvar> <ci> y </ci> </bvar>
    <bvar> <ci> x </ci> </bvar>
    <degree> <cn> 4 </cn> </degree>
    <ci type="fn"> f </ci>
  </apply>
</math>

```

$$\frac{\partial^k f(x,y)}{\partial x^m \partial y^n}$$

```

<math>
  <apply> <partialdiff/>
    <bvar>
      <ci> x </ci> <degree> <ci> m </ci> </degree>
    </bvar>
    <bvar>
      <ci> y </ci> <degree> <ci> n </ci> </degree>
    </bvar>
    <degree> <ci> k </ci> </degree>
    <apply> <ci type="fn"> f </ci>
      <ci> x </ci>
      <ci> y </ci>
    </apply>

```

```
</apply>
</math>
```

$$\frac{\partial^{m+n} f(x,y)}{\partial x^m \partial y^n}$$

```
<math>
  <apply> <partialdiff/>
    <bvar>
      <ci> x </ci> <degree> <ci> m </ci> </degree>
    </bvar>
    <bvar>
      <ci> y </ci> <degree> <ci> n </ci> </degree>
    </bvar>
    <apply> <ci type="fn"> f </ci>
      <ci> x </ci>
      <ci> y </ci>
    </apply>
  </apply>
</math>
```

When a degree is not specified, it is deduced from the context, but since this is not 100% watertight, you can best be complete in your specification.

These examples are taken from the MATHML specification. In the example document that comes with this manual you can find a couple of more.

<> *fn* There are a lot of predefined functions and operators. If you want to introduce a new one, the *fn* element can be used. In the following example we have turned the \pm and \mp symbols into (coupled) operators.

$$x \pm 1 \ x \mp 1 = x^2 - 1$$

```
<math>
  <apply> <eq/>
    <apply> <times/>
      <apply> <fn> <ci> &plusminus; </ci> </fn>
        <ci> x </ci>
        <cn> 1 </cn>
      </apply>
    </apply>
  </math>
```

```

    <apply> <fn> <ci> &minusplus; </ci> </fn>
      <ci> x </ci>
      <cn> 1 </cn>
    </apply>
  </apply>
  <apply> <minus/>
    <apply> <power/>
      <ci> x </ci>
      <cn> 2 </cn>
    </apply>
    <cn> 1 </cn>
  </apply>
</math>

```

The typeset result depends on the presence of a handler, which in this case happens to be true.

<> *matrix, matrixrow* Matrices are one of the building blocks of linear algebra and therefore both presentational and content MATHML have dedicated elements for defining them.

$$\begin{pmatrix} 23 & 87 & c \\ 41 & b & 33 \\ a & 65 & 16 \end{pmatrix}$$

```

<math>
  <matrix>
    <matrixrow> <cn> 23 </cn> <cn> 87 </cn> <ci> c </ci> </matrixrow>
    <matrixrow> <cn> 41 </cn> <ci> b </ci> <cn> 33 </cn> </matrixrow>
    <matrixrow> <ci> a </ci> <cn> 65 </cn> <cn> 16 </cn> </matrixrow>
  </matrix>
</math>

```

<> *vector* We make a difference between a vector specification and a vector variable. A specification is presented as a list:

$$(x,y)$$

```

<math>
  <vector>
    <ci> x </ci>
    <ci> y </ci>
  </vector>
</math>

```

When the *vector* element has one child element, we use a right arrow to identify the variable as vector.

$$\vec{A} \times \vec{B}$$

```

<math>
  <apply> <vectorproduct/>
    <vector> <ci> A </ci> </apply>
    <vector> <ci> B </ci> </apply>
  </vector>
</math>

```

<> *grad, curl, ident, divergence* These elements expand into named functions, but we can imagine that in the future a more appropriate visualization will be provided as an option.

$$\text{grad } A \neq \text{curl } B \neq \text{id } C \neq \text{div } D$$

```

<math>
  <apply> <neq/>
    <apply> <grad/>      <ci> A </ci> </apply>
    <apply> <curl/>      <ci> B </ci> </apply>
    <apply> <ident/>     <ci> C </ci> </apply>
    <apply> <divergence/> <ci> D </ci> </apply>
  </apply>
</math>

```

<> *lambda, bvar* The lambda specification of a function needs a *bvar* element. The visualization can be influenced with processing instructions as described in a later chapter.

$$x \mapsto \sin\left(x - \frac{x}{2}\right)$$

```

<math>
  <lambda>
    <bvar> <ci> x </ci> </bvar>
    <apply> <sin/>
      <apply> <minus/>
        <ci> x </ci>
        <apply> <divide/>
          <ci> x </ci>
          <cn> 2 </cn>
        </apply>
      </apply>
    </apply>
  </lambda>
</math>

```

<> *piecewise, piece, otherwise* There are not so many elements that deal with combinations of formulas or conditions. The *piecewise* is the only real selector available. The following example defines how the state of n depends on the state of x .

$$n = \begin{cases} -1 & x < 0 \\ 1 & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

```

<math>
  <apply> <eq/>
    <ci> n </ci>
    <piecewise>
      <piece>
        <apply> <minus/>
          <cn> 1 </cn>
        </apply>
        <apply> <lt/>
          <ci> x </ci>
          <cn> 0 </cn>
        </apply>
      </piece>
      <piece>
        <cn> 1 </cn>
        <apply> <gt/>

```



```

      <ci> x </ci>
      <cn> 0 </cn>
    </apply>
  </piece>
  <otherwise>
    <cn> 0 </cn>
  </otherwise>
</piecewise>
</apply>
</math>

```

We could have use a third *piece* instead of (optional) *otherwise*.

<> *forall, exists, condition* Conditions are often used in combination with elements like *forall*. There are several ways to convert and combine them in formulas and environments, so you may expect more alternatives in the future.

$$\forall_x x < 9 \mid x < 10$$

```

<math>
  <apply> <forall/>
    <bvar> <ci> x </ci> </bvar>
    <condition>
      <apply> <lt/>
        <ci> x </ci>
        <cn> 9 </cn>
      </apply>
    </condition>
    <apply> <lt/>
      <ci> x </ci>
      <cn> 10 </cn>
    </apply>
  </apply>
</math>

```

The next example is taken from the specifications with a few small changes.

$$\forall_x x \in \mathbb{N} \mid \exists_{p,q} p \in \mathbb{P} \wedge q \in \mathbb{P} \wedge p + q = 2x$$

```

<math>
  <apply> <forall/>
    <bvar> <ci> x </ci> </bvar>
    <condition>
      <apply> <in/>
        <ci> x </ci>
        <ci type="set"> N </ci>
      </apply>
    </condition>
  <apply> <exists/>
    <bvar> <ci> p </ci> </bvar>
    <bvar> <ci> q </ci> </bvar>
    <condition>
      <apply> <and/>
        <apply> <in/>
          <ci> p </ci>
          <ci type="set"> P </ci>
        </apply>
        <apply> <in/>
          <ci> q </ci>
          <ci type="set"> P </ci>
        </apply>
      <apply> <eq/>
        <apply> <plus/> <ci> p </ci> <ci> q </ci> </apply>
        <apply> <times/> <cn> 2 </cn> <ci> x </ci> </apply>
      </apply>
    </condition>
  </apply>
</math>

```

<> *factorof, tendsto* The *factorof* element is applied to its two child elements and contrary to most functions, the symbol is placed between the elements instead of in front.

$$a \mid b$$

```

<math>
  <apply> <factorof/>

```

```

    <ci> a </ci>
    <ci> b </ci>
  </apply>
</math>

```

The same is true for the *tendsto* element.

$$a \rightarrow b$$

```

<math>
  <apply> <tendsto/>
    <ci> a </ci>
    <ci> b </ci>
  </apply>
</math>

```

<> *compose* This is a nasty element since it has to take care of braces in special ways and therefore has to analyse its child elements.

$$f \circ g \circ h$$

```

<math>
  <apply> <compose/>
    <ci type="fn"> f </ci>
    <ci type="fn"> g </ci>
    <ci type="fn"> h </ci>
  </apply>
</math>

```

$$(f \circ g) x$$

```

<math>
  <apply>
    <apply> <compose/>
      <fn> <ci> f </ci> </fn>
      <fn> <ci> g </ci> </fn>
    </apply>
    <ci> x </ci>
  </apply>
</math>

```

<> *laplacian* A laplacian function is typeset using a ∇ (nabla) symbol.

$$\nabla^2 x$$

```
<math>
  <apply> <laplacian/>
    <ci> x </ci>
  </apply>
</math>
```

<> *mean, sdev, variance, median, mode* When statistics shows up in math text books, the *sum* element is likely to show up, probably in combination with the *for* for statistics meaningful symbolic representation of variables. The mean value of a series of observations is defined as:

$$\bar{x} = \frac{\sum x}{n}$$

```
<math>
  <apply> <eq/>
    <apply> <mean/>
      <ci> x </ci>
    </apply>
    <apply> <divide/>
      <apply> <sum/>
        <ci> x </ci>
      </apply>
      <ci> n </ci>
    </apply>
  </apply>
</math>
```

or more beautiful:

$$\bar{x} = \frac{1}{n} \sum x$$

```
<math>
  <apply> <eq/>
    <apply> <mean/>
      <ci> x </ci>
```

```

</apply>
<apply> <times/>
  <apply> <divide/>
    <cn> 1 </cn>
    <ci> n </ci>
  </apply>
  <apply> <sum/>
    <ci> x </ci>
  </apply>
</apply>
</math>

```

Of course this definition is not that perfect, but we will present a better alternative in the chapter on combined markup. The definition of the standard deviation is more complicated:

$$\sigma(x) \approx \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

```

<math>
  <apply> <approx/>
    <apply> <sdev/>
      <ci> x </ci>
    </apply>
    <apply> <root/>
      <apply> <divide/>
        <apply> <sum/>
          <apply> <power/>
            <apply> <minus/>
              <ci> x </ci>
            </apply>
            <apply> <mean/>
              <ci> x </ci>
            </apply>
          </apply>
        </apply>
        <cn> 2 </cn>
      </apply>
    </apply>
    <apply> <minus/>

```

```

      <ci> n </ci>
      <cn> 1 </cn>
    </apply>
  </apply>
</apply>
</math>

```

The next example demonstrates the usage of the *variance* in its own definition.

$$\sigma(x)^2 = \overline{(x - \bar{x})^2} \approx \frac{1}{n-1} \sum (x - \bar{x})^2$$

```

<math>
  <apply> <eq/>
    <apply> <variance/>
      <ci> x </ci>
    </apply>
    <apply> <approx/>
      <apply> <mean/>
        <apply> <power/>
          <apply> <minus/>
            <ci> x </ci>
          <apply> <mean/>
            <ci> x </ci>
          </apply>
        </apply>
        <cn> 2 </cn>
      </apply>
    </apply>
    <apply> <times/>
      <apply> <divide/>
        <cn> 1 </cn>
      <apply> <minus/>
        <ci> n </ci>
        <cn> 1 </cn>
      </apply>
    </apply>
    <apply> <sum/>
      <apply> <power/>

```

```

    <apply> <minus/>
      <ci> x </ci>
      <apply> <mean/>
        <ci> x </ci>
      </apply>
    </apply>
    <cn> 2 </cn>
  </apply>
</math>

```

The *median* and *mode* of a series of observations have no special symbols and are presented as is.

- <> *moment, momentabout, degree* Because MATHML is used for a wide range of applications, there can be information in a definition that does not end up in print but is only used in some cases. This is illustrated in the next example.

$$\langle X^3 \rangle$$

```

<math>
  <apply> <moment/>
    <degree>
      <cn> 3 </cn>
    </degree>
    <momentabout>
      <ci> p </ci>
    </momentabout>
    <ci> X </ci>
  </apply>
</math>

```

- <> *determinant, transpose* These two (and the following) are used to manipulate matrices, either or not in a symbolic way. A simple determinant or transpose looks like:

$$|A|$$

```
<math>
  <apply> <determinant/>
    <ci type="matrix"> A </ci>
  </apply>
</math>
```

$$A^T$$

```
<math>
  <apply> <transpose/>
    <ci type="matrix"> A </ci>
  </apply>
</math>
```

When the *determinant* element is applied to full blown matrix, the braces are omitted and replaced by the vertical bars.

$$|I| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

```
<math>
  <apply> <eq/>
    <apply> <determinant/>
      <ci> I </ci>
    </apply>
    <apply> <determinant/>
      <matrix>
        <matrixrow> <cn> 1 </cn> <cn> 0 </cn> </matrixrow>
        <matrixrow> <cn> 0 </cn> <cn> 1 </cn> </matrixrow>
      </matrix>
    </apply>
    <cn> 1 </cn>
  </apply>
</math>
```

<> *selector* The *selector* element can be used to index a matrix cell or variable. This element honors the braces.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}_1$$

```
<math>
  <apply> <selector/>
    <matrix>
      <matrixrow> <cn> 1 </cn> <cn> 2 </cn> </matrixrow>
      <matrixrow> <cn> 3 </cn> <cn> 4 </cn> </matrixrow>
    </matrix>
    <cn> 1 </cn>
  </apply>
</math>
```

A more common usage of the selector is the following:

$$x_i$$

```
<math>
  <apply> <selector/>
    <ci> x </ci>
    <ci> i </ci>
  </apply>
</math>
```

It is possible to pass a comma separated list of indices:

$$x_{1,2}$$

```
<math>
  <apply> <selector/>
    <ci> x </ci> <cn> 1,2 </cn>
  </apply>
</math>
```

If you want to have a more verbose index, you can use the *csymbol* element, flagged with text enciding.

$$x_{\max}$$

```
<math>
  <apply> <selector/>
```

```

    <ci> x </ci>
    <csymbol encoding="text"> max </csymbol>
  </apply>
</math>

```

<> *card* A cardinality is visualized using vertical bars. [But what exactly is meant with cardinality?]

$$|A| = 5$$

```

<math>
  <apply> <eq/>
    <apply> <card/>
      <ci> A </ci>
    </apply>
    <ci> 5 </ci>
  </apply>
</math>

```

<> *domain, codomain, image* The next couple of examples are taken from the MATHML specification and demonstrate the usage of the not that spectacular domain related elements.

$$\text{domain}(f) = \mathbb{R}$$

```

<math>
  <apply> <eq/>
    <apply> <domain/>
      <fn> <ci> f </ci> </fn>
    </apply>
    <reals/>
  </apply>
</math>

```

These are typically situations where the *fn* element may show up.

$$\text{codomain}(f) = \mathbb{Q}$$

```

<math>
  <apply> <eq/>
    <apply> <codomain/>
      <fn> <ci> f </ci> </fn>
    </apply>
  </apply>
</math>

```

This example from the MATHML specification demonstrates a typical usage of the *image* element. As with the previous two, it is applied to a function, in this case the predefined *sin*.

$$\text{image}(\sin) = [-1, 1]$$

```

<math>
  <apply> <eq/>
    <apply> <image/>
      <sin/>
    </apply>
    <interval>
      <cn> -1 </cn>
      <cn> 1 </cn>
    </interval>
  </apply>
</math>

```

<> *domainofapplication* This is another seldom used element. Actually, this element is a further specification of the outer level applied function.

$$\int_C f$$

```

<math>
  <apply> <int/>
    <domainofapplication>
      <ci> C </ci>
    </domainofapplication>
    <ci> f </ci>
  </apply>
</math>

```

```

</apply>
</math>

```

<> *semantics, annotation, annotation-xml* We will never know what Albert Einstein would have thought about MATHML. But we do know for sure that coding one of his famous findings in XML takes much more tokens than it takes in T_EX.

Within a *semantics* element there can be many *annotation* elements. When using CON_TE_XT, the elements that can be identified as being encoded in T_EX will be treated as such. Currently, the related *annotation-xml* element is ignored.

$$e = mc^2$$

```

<math>
  <semantics>
    <apply> <eq/>
      <ci> e </ci>
      <apply> <times/>
        <ci> m </ci>
        <apply> <power/>
          <ci> c </ci>
          <cn> 2 </cn>
        </apply>
      </apply>
    </apply>
    <annotation encoding="TeX">
      e = m c^2
    </annotation>
  </semantics>
</math>

```

<> *integers, reals, ...* Sets are characterized with special (often blackboard) symbols. These symbols are not always available.

integers	Z
reals	R
rational	Q
natural numbers	N

complexes	C
primes	P

- <> *pi, imaginaryi, exponentiale* Being a greek character, π is a distinctive character. In most math documents the imaginary i and exponential e are typeset as any math identifier.

pi	π
imaginaryi	i
exponentiale	e

- <> *eulergamma, infinity, emptyset* There are a couple of more special tokens. As with the other ones, they can be changed by reassigning the corresponding entities.

eulergamma	γ
infinity	∞
emptyset	\emptyset

- <> *notanumber* Because MATHML is used for more purposes than typesetting, there are a couple of elements that do not make much sense in print. One of these is *notanumber*, which is issued by programs as error code or string.

$$\frac{x}{0} = \text{NaN}$$

```
<math>
  <apply> <eq/>
    <apply> <divide/>
      <ci> x </ci>
      <cn> 0 </cn>
    </apply>
  <notanumber/>
</apply>
</math>
```

- <> *true, false* When assigning to a boolean variable, or in boolean expressions one can use 0 or 1 to identify the states, but if you want to be more verbose, you can use these elements.

$$1_2 \equiv \text{true}$$

```
<math>
  <apply> <equivalent/>
    <cn type="integer" base="2"> 1 </cn>
    <true/>
  </apply>
</math>
```

<> *declare* Reusing definitions would be a nice feature, but for the moment the formal specification of this element currently does not give us the freedom to use it the way we want.

declare A as (a,b,c)

```
<math>
  <declare>
    <ci> A </ci>
    <vector>
      <ci> a </ci>
      <ci> b </ci>
      <ci> c </ci>
    </vector>
  </declare>
</math>
```

<> *csymbol* This element will be implemented as soon as I have an application for it.

Mixed markup

The advantage of presentational markup is that you can build complicated formulas using super- and subscripts and other elements. The drawback is that the look and feel is rather fixed and cannot easily be adapted to the purpose that the document serves. Take for instance the difference between

$$\log_2 x$$

and

$$^2\log x$$

Both formulas were defined in content MATHML, so no explicit super- and subscripts were used. In the next chapter we will see how to achieve such different appearances.

There are situations where content MATHML is not rich enough to achieve the desired output. This omission in content MATHML forces us to fall back on presentational markup.

$$P_1 = P_2 = 1.01 \approx 1$$

Here we used presentational elements inside a content *ci* element. We could have omitted the outer *ci* element, but since the content MATHML parser may base its decisions on the content elements it finds, it is best to keep the outer element there.

```
<math>
  <apply> <eq/>
    <ci> <msub> <mi> P </mi> <mi> 1 </mi> </msub> </ci>
    <ci> <msub> <mi> P </mi> <mi> 2 </mi> </msub> </ci>
    <apply> <approx/>
      <cn> 1.01 </cn>
      <cn> 1 </cn>
    </apply>
  </apply>
</math>
```

The lack of an index element can be quite prominent. For instance, when in an expose about rendering we want to explore the mapping from coordinates in user space to those in device space, we use the following formula.

$$(D_x, D_y, 1) = (U_x, U_y, 1) \begin{pmatrix} s_x & r_x & 0 \\ r_y & s_y & 0 \\ t_x & t_y & 1 \end{pmatrix}$$

```

<math>
  <apply> <eq/>
    <vector>
      <ci> <msub> <mi> D </mi> <mi> x </mi> </msub> </ci>
      <ci> <msub> <mi> D </mi> <mi> y </mi> </msub> </ci>
      <cn> 1 </cn>
    </vector>
    <apply> <times/>
      <vector>
        <ci> <msub> <mi> U </mi> <mi> x </mi> </msub> </ci>
        <ci> <msub> <mi> U </mi> <mi> y </mi> </msub> </ci>
        <cn> 1 </cn>
      </vector>
      <matrix>
        <matrixrow>
          <ci> <msub> <mi> s </mi> <mi> x </mi> </msub> </ci>
          <ci> <msub> <mi> r </mi> <mi> x </mi> </msub> </ci>
          <cn> 0 </cn>
        </matrixrow>
        <matrixrow>
          <ci> <msub> <mi> r </mi> <mi> y </mi> </msub> </ci>
          <ci> <msub> <mi> s </mi> <mi> y </mi> </msub> </ci>
          <cn> 0 </cn>
        </matrixrow>
        <matrixrow>
          <ci> <msub> <mi> t </mi> <mi> x </mi> </msub> </ci>
          <ci> <msub> <mi> t </mi> <mi> y </mi> </msub> </ci>
          <cn> 1 </cn>
        </matrixrow>
      </matrix>
    </apply>
  </apply>
</math>

```

Again, the *msub* element provides a way out, as in the next examples, which are adapted versions of formulas we used when demonstrating the statistics related elements.

$$\bar{x} = \frac{1}{n} \sum_i x$$


```

<math>
  <apply> <eq/>
    <apply> <mean/>
      <ci> x </ci>
    </apply>
    <apply> <times/>
      <apply> <divide/>
        <cn> 1 </cn>
        <ci> n </ci>
      </apply>
      <apply> <sum/>
        <bvar> <ci> i </ci> </bvar>
        <ci> x </ci>
      </apply>
    </apply>
  </math>

```

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x$$

```

<math>
  <apply> <eq/>
    <apply> <mean/>
      <ci> x </ci>
    </apply>
    <apply> <times/>
      <apply> <divide/>
        <cn> 1 </cn>
        <ci> n </ci>
      </apply>
      <apply> <sum/>
        <bvar> <ci> i </ci> </bvar>
        <lowlimit> <cn> 1 </cn> </lowlimit>
        <uplimit> <cn> n </cn> </uplimit>
        <ci> x </ci>
      </apply>
    </apply>
  </math>

```

</apply>
</math>

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

<math>
 <apply> <eq/>
 <apply> <mean/>
 <ci> x </ci>
 </apply>
 <apply> <times/>
 <apply> <divide/>
 <cn> 1 </cn>
 <ci> n </ci>
 </apply>
 <apply> <sum/>
 <bvar> <ci> i </ci> </bvar>
 <lowlimit> <cn> 1 </cn> </lowlimit>
 <uplimit> <cn> n </cn> </uplimit>
 <ci> <msub> <mi> x </mi> <mi> i </mi> </msub> </ci>
 </apply>
 </apply>
</math>

Directives

Some elements can be tuned by changing their attributes. Especially when formulas are defined by a team of people or when they are taken from a repository, there is a good chance that inconsistencies will show up.

In `CONTEXT`, you can influence the appearance by setting the typesetting parameters of (classes of) elements. You can do this either by adding processing instructions, or by using the `CONTEXT` command `\setupMMLappearance`. Although the first method is more in the spirit of XML, the second method is more efficient and consistent. As a processing instruction, a directive looks like:

```
<?context-mathml-directive element key value ?>
```

This is equivalent to the `CONTEXT` command:

```
\setupMMLappearance [element] [key=value]
```

Some settings concern a group of elements, in which case a group classification (like `sign`) is used.

<> *scripts* By default, nested super- and subscripts are kind of isolated from each other. If you want a combined script, there is the *msubsup*. You can however force combinations with a directive.

$$x_1^2$$

$$x_1^2$$

```
<math>
  <msup>
    <msub> <mi> x </mi> <mn> 1 </mn> </msub>
    <mn> 2 </mn>
  </msup>
</math>
<?context-mathml-directive scripts alternative b ?>
<math>
  <msup>
    <msub> <mi> x </mi> <mn> 1 </mn> </msub>
    <mn> 2 </mn>
  </msup>
</math>
```

- <> *sign* The core element of MATHML is *apply*. Even simple formulas will often have more than one (nested) *apply*. The most robust way to handle nested formulas is to use braces around each sub formula. No matter how robust this is, when presented in print we want to use as less braces as possible.

$$7 + 5 - 3$$

This expression shows addition as well as subtraction.

```
<math>
  <apply> <plus/>
    <cn> 7 </cn>
    <cn> 5 </cn>
  <apply> <minus/>
    <cn> 3 </cn>
  </apply>
</math>
```

In principle subtraction is adding negated numbers, so it would have been natural to have just an addition (*plus*) and negation operator. However, MATHML provides both a *plus* and *minus* operator, where the latter can be used as a negation. So in fact we have:

$$7 + 5 + (-3)$$

Now imagine that a teacher wants to stress this negation in the way presented here, using parentheses. Since all the examples shown here are typeset directly from the MATHML source, you may expect a solution, so here it is:

```
<math>
  <?context-mathml-directive sign reduction no ?>
  <apply> <plus/>
    <cn> 7 </cn>
    <cn> 5 </cn>
  <apply> <minus/>
    <cn> 3 </cn>
  </apply>
</math>
```

By default signs are reduced, but one can disable that at the document and/or formula level using a processing instruction at the top of the formula. There are of course circumstances where the parentheses cannot be left out.

$$a + b + c + d$$

```
<math>
  <apply> <plus/>
    <ci> a </ci>
    <apply> <plus/> <ci> b </ci> <ci> c </ci> </apply>
    <ci> d </ci>
  </apply>
</math>
```

$$a - (b - c) - d$$

```
<math>
  <apply> <minus/>
    <ci> a </ci>
    <apply> <minus/> <ci> b </ci> <ci> c </ci> </apply>
    <ci> d </ci>
  </apply>
</math>
```

$$a + b - c + d$$

```
<math>
  <apply> <plus/>
    <ci> a </ci>
    <apply> <minus/> <ci> b </ci> <ci> c </ci> </apply>
    <ci> d </ci>
  </apply>
</math>
```

$$a - (b + c) - d$$

```
<math>
  <apply> <minus/>
    <ci> a </ci>
    <apply> <plus/> <ci> b </ci> <ci> c </ci> </apply>
  </apply>
</math>
```

```

    <ci> d </ci>
  </apply>
</math>

```

Another place where parentheses are not needed is the following:

```

<math>
  <apply> <minus/>
    <apply> <exp/>
      <cn> 3 </cn>
    </apply>
  </apply>
</math>

```

This means that the interpreter of this kind of MATHML has to analyze child elements in order to choose the right way to typeset the formula. The output will look like:

$$-e^3$$

By default, as less braces as possible are used. As demonstrated, a special case is when *plus* and *minus* have one sub element to deal with. If you really want many braces there, you can turn off sign reduction.

sign	reduction	yes	use as less braces as possible
		no	always use braces

We will demonstrate these alternatives with an example.

$$a + \sin b + c^5 + \sin^2 d + e$$

We need quite some code to encode this formula.

```

<math>
  <apply> <plus/>
    <ci> a </ci>
    <apply> <sin/>
      <ci> b </ci>
    </apply>
    <apply> <power/>
      <ci> c </ci>

```

```

    <cn> 5 </cn>
  </apply>
<apply> <power/>
  <apply> <sin/>
    <ci> d </ci>
  </apply>
  <cn> 2 </cn>
</apply>
<ci> e </ci>
</apply>
</math>

```

With power reduction turned off, we get:

$$a + \sin b + c^5 + (\sin d)^2 + e$$

As directive we used:

```
<?context-mathml-directive power reduction no ?>
```

The following example illustrates that we should be careful in coding such formulas; here the *power* is applied to the argument of *sin*.

$$a + \sin b + c^5 + \sin(d^2) + e$$

```

<math>
  <apply> <plus/>
    <ci> a </ci>
    <apply> <sin/>
      <ci> b </ci>
    </apply>
    <apply> <power/>
      <ci> c </ci>
      <cn> 5 </cn>
    </apply>
    <apply> <sin/>
      <apply> <power/>
        <ci> d </ci>
        <cn> 2 </cn>
      </apply>
    </apply>
    <ci> e </ci>
  </apply>
</math>

```

```

    </apply>
    <ci> e </ci>
  </apply>
</math>

```

<> *divide* Divisions can be very space consuming but there is a way out: using a forward slash symbol. You can set the level at which this will take place. By default, fractions are typeset in the traditional way.

$$\frac{1}{1 + \frac{1}{x}}$$

```

<math>
  <apply> <divide/>
    <cn> 1 </cn>
    <apply> <plus/>
      <cn> 1 </cn>
      <apply> <divide/>
        <cn> 1 </cn>
        <ci> x </ci>
      </apply>
    </apply>
  </apply>
</math>

```

$$\frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}$$

```

<math>
  <apply> <divide/>
    <cn> 1 </cn>
    <apply> <plus/>
      <cn> 1 </cn>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <plus/>
          <cn> 1 </cn>
          <apply> <divide/>
            <cn> 1 </cn>

```



```

        <ci> x </ci>
      </apply>
    </apply>
  </apply>
</math>

```

$$\frac{1}{1 + 1/x}$$

$$\frac{1}{1 + 1/(1 + 1/x)}$$

```
<?context-mathml-directive divide level 1?>
```

$$\frac{1}{1 + \frac{1}{x}}$$

$$\frac{1}{1 + \frac{1}{1 + 1/x}}$$

```
<?context-mathml-directive divide level 2?>
```

<> *relation* You should keep in mind that (at least level 2) content MATHML is not that rich in terms of presenting your ideas in a visually attractive way. On the other hand, because the content is highly structured, some intelligence can be applied when typesetting them. By default, a relation is not vertically aligned but typeset horizontally.

If an application just needs raw formulas, definitions like the following are all right.

```

<math>
  <apply> <eq/>
    <apply> <plus/>
      <ci> a </ci>
      <ci> b </ci>
      <ci> c </ci>
    </apply>
    <apply> <plus/>
      <ci> d </ci>
      <ci> e </ci>
    </apply>
  </math>

```

```

<apply> <plus/>
  <ci> f </ci>
  <ci> g </ci>
  <ci> h </ci>
  <ci> i </ci>
</apply>
<cn> 123 </cn>
</apply>
</math>

```

The typeset result will bring no surprises:

$$a + b + c = d + e = f + g + h + i = 123$$

But, do we want to show a formula that way? And what happens with much longer formulas? You can influence the appearance with processing instructions.

relation	align	no	don't align relations
		left	align all relations left
		right	align all relations right
		first	place the leftmost relation left
		last	place the rightmost relation right

The next couple of formulas demonstrate in what way the previously defined formula is influenced by the processing instructions.

$$\begin{aligned}
 a + b + c &= \\
 &= d + e \\
 &= f + g + h + i = \\
 &123
 \end{aligned}$$

<?context-mathml-directive relation align left ?>

$$\begin{aligned}
 &a + b + c \\
 &= d + e \\
 &= f + g + h + i \\
 &= 123
 \end{aligned}$$

<?context-mathml-directive relation align right ?>

$$\begin{aligned}
 a + b + c &= d + e \\
 &= f + g + h + i \\
 &= 123
 \end{aligned}$$

<?context-mathml-directive relation align first ?>

$$\begin{aligned}
 a + b + c &= \\
 d + e &= \\
 f + g + h + i &= 123
 \end{aligned}$$

<?context-mathml-directive relation align last ?>

- <> *base* When in a document several number systems are used, it can make sense to mention the base of the number. There are several ways to identify the base.

base	symbol	numbers	a (decimal) number
		characters	one character
		text	a mnemonic
		no	no symbol

By default, when specified, a base is identified as number.

```

<math>
  <cn type="integer" base="8"> 1427 </cn>
</math>

```

1427₈

<?context-mathml-directive base symbol numbers ?>

1427₀

<?context-mathml-directive base symbol characters ?>

1427_{OCT}

<?context-mathml-directive base symbol text ?>

- <> *function* There is a whole bunch of functions available as empty element, like *sin* and *log*. When a function is applied to a function, braces make not much sense and placement is therefore disabled.

function	reduction	yes	chain functions without braces
		no	put braces around nested functions

```
<math>
  <apply> <sin/> <ci> x </ci> </apply>
</math>
```

$$\sin x$$

```
<?context-mathml-directive function reduction yes?>
```

$$\sin(x)$$

```
<?context-mathml-directive function reduction no?>
```

<> *limits* When limits are placed on top of the limitation symbol, this generally looks better than when they are placed alongside. You can also influence limit placement per element. This feature is available for *int*, *sum*, *product* and *limit*.

limit	location	top	place limits on top of the symbols
		right	attached limits as super/subscripts

```
<math>
  <apply> <int/>
    <bvar> <ci> x </ci> </bvar>
    <lowlimit> <cn> 0 </cn> </lowlimit>
    <uplimit> <cn> 1 </cn> </uplimit>
  </apply>
</math>
```

$$\int_0^1 dx$$

```
<?context-mathml-directive int location top?>
```

$$\int_0^1 dx$$

```
<?context-mathml-directive int location right?>
```

- <> *declare* Currently declarations are not supposed to end up in print. By default we typeset a message, but you can as well completely hide declarations.

```
declare state start show declarations
                stop  ignore (hide) declarations
```

- <> *lambda* There is more than one way to visualize a lambda function. As with some other settings, changing the appearance can best take place at the document level.

```
lambda alternative b show lambda as arrow
                  a show lambda as set
```

```
<math>
  <lambda>
    <bvar> <ci> x </ci> </bvar>
    <apply> <log/>
      <ci> x </ci>
    </apply>
  </lambda>
</math>
```

$$\lambda(x, \log x)$$

```
<?context-mathml-directive lambda alternative a?>
```

$$x \mapsto \log x$$

```
<?context-mathml-directive lambda alternative b?>
```

- <> *power* Taking the power of a function looks clumsy when braces are put around the function. Therefore, by default, the power is applied to the function symbol instead of the whole function.

```
power reduction yes attach symbol to function symbol
                no  attach symbol to function argument
```

```
<math>
  <apply> <power/>
    <apply> <ln/>
```

```

    <ci> x </ci>
  </apply>
  <cn> 3 </cn>
</apply>
</math>

```

$$\ln^3 x$$

```
<?context-mathml-directive power reduction yes?>
```

$$(\ln x)^3$$

```
<?context-mathml-directive power reduction no?>
```

<> *diff* Covering all kind of differential formulas is not trivial. Currently we support two locations for the operand (function). By default the operand is placed above the division line.

diff	location	top	put the operand in the fraction
		right	put the operand after the fraction

```

<math>
  <apply> <diff/>
    <bvar>
      <ci> x </ci>
      <degree> <cn> 2 </cn> </degree>
    </bvar>
    <apply> <fn> <ci> f </ci> </fn>
      <apply> <plus/>
        <apply> <times/>
          <cn> 2 </cn>
          <ci> x </ci>
        </apply>
        <cn> 1 </cn>
      </apply>
    </apply>
  </apply>
</math>

```

$$\frac{d^2 f(2x + 1)}{dx^2}$$

<?context-mathml-directive diff location top?>

$$\frac{d^2}{dx^2} (f(2x + 1))$$

<?context-mathml-directive diff location right?>

<> *vector* Depending on the complication of a vector or on the available space, you may wish to typeset a vector horizontally or vertically. By default a vector is typeset horizontally.

vector	direction	horizontal	put vector elements alongside
		vertical	stack vector elements

```
<math>
  <apply> <eq/>
    <vector>
      <ci> x </ci>
      <ci> y </ci>
      <ci> z </ci>
    </vector>
    <vector>
      <cn> 1 </cn>
      <cn> 0 </cn>
      <cn> 1 </cn>
    </vector>
  </apply>
</math>
```

$$(x,y,z) = (1,0,1)$$

<?context-mathml-directive vector direction horizontal?>

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

<?context-mathml-directive vector direction vertical?>

- <> *times* Depending on the audience, a multiplication sign is implicit (absent) or represented by a regular times symbol or a dot.

times	symbol	no	don't add a symbol
		yes	separate operands by a times (×)
		dot	separate operands by a dot (·)

```
<math>
  <apply> <plus/>
    <ci> x </ci>
    <apply> <times/>
      <cn> 2 </cn>
      <ci> x </ci>
    </apply>
  </apply>
</math>
```

$$x + 2x$$

```
<?context-mathml-directive times symbol no?>
```

$$x + 2 \times x$$

```
<?context-mathml-directive times symbol yes?>
```

$$x + 2 \cdot x$$

```
<?context-mathml-directive times symbol dot?>
```

- <> *log* The location of a logbase depends on tradition and/or preference, which is why we offer a few alternatives: as pre superscript (in the right top corner before the symbol) or as post subscript (in the lower left corner after the symbol).

log	location	right	place logbase at the right top
		left	place logbase at the lower left

```
<math>
  <apply> <log/>
    <logbase>
      <ci> 3 </ci>
    </logbase>
  </apply>
</math>
```



```

</logbase>
<apply> <plus/>
  <ci> x </ci>
  <cn> 1 </cn>
</apply>
</apply>
</math>

```

$$\log_3 (x + 1)$$

```
<?context-mathml-directive log location right?>
```

$${}^3\log (x + 1)$$

```
<?context-mathml-directive log location left?>
```

<> *polar* The location of a logbase depends on tradition and/or preference, which is why we offer a few alternatives: as pre superscript (in the right top corner before the symbol) or as post subscript (in the lower left corner after the symbol).

polar	alternative	a	explicit polar notation
		b	exponential power notation
		c	exponential function notation

```

<math>
  <cn type="polar"> 2 <sep/> <pi/> </cn>
</math>

```

$$\text{Polar } (2, \pi)$$

```
<?context-mathml-directive polar alternative a?>
```

$$2e^{\pi i}$$

```
<?context-mathml-directive polar alternative b?>
```

$$2 \exp (\pi i)$$

```
<?context-mathml-directive polar alternative c?>
```

<> *e-notation* Depending on the context, you may want to typeset the number 1.23e4 not as this sequence, but using a multiplier construct. As with the *times*, we support both multiplication symbols.

enotation	symbol	no	no interpretation
		yes	split exponent, using \times
		dot	split exponent, using \cdot

```
<math>
  <cn type="e-notation">10<sep/>23</cn>
</math>
```

$$10e23$$

```
<?context-mathml-directive enotation symbol no?>
```

$$10 \times 10^{23}$$

```
<?context-mathml-directive enotation symbol yes?>
```

$$10 \cdot 10^{23}$$

```
<?context-mathml-directive enotation symbol dot?>
```

Features

In this chapter we will introduce some additional elements that are not part of MATHML, but can be used to organize your math in such a way that it can be part of elementary math books.

Typesetting modes

Math can be typeset in line or in display. In order not to widen up the text of a paragraph too much, inline math is typeset more cramped. Since MATHML does provide just a general purpose *math* element we have to provide the information needed using other elements. Consider the following text.

To what extent is math supposed to reflect the truth and nothing but the truth? Consider the simple expression $10 = 3 + 7$. Many readers will consider this the truth, but then, can we assume that the decimal notation is used?

$$10 = 3 + x$$

In many elementary math books, you can find expressions like the previous. Because in our daily life we use the decimal numbering system, we can safely assume that $x = 7$. But, without explicitly mentioning this boundary condition, more solutions are correct.

$$10 = 3 + 5 \tag{1a}$$

In formula 1a we see an at first sight wrong formula. But, if we tell you that octal numbers are used, your opinion may change instantly. A rather clean way out of this confusion is to extend the notation of numbers by explicitly mentioning the base.

$$10_8 = 3_8 + 5_8 \tag{1b}$$

Of course, when a whole document is in octal notation, a proper introduction is better than annotated numbers as used in formula 1b.

In terms of XML this can look like:

To what extent is math supposed to reflect the truth and nothing but the truth? Consider the simple expression `<math> <apply> <eq/> <cn> 10 </cn> <apply> <plus/> <cn> 3 </cn> <cn> 7 </cn> </apply> </apply> </math>`. Many readers will consider this the truth, but then, can we assume that the decimal notation is used?

```
<formula>
  <math>
    <apply> <eq/>
      <cn> 10 </cn>
```

```

    <apply> <plus/>
      <cn> 3 </cn>
      <ci> x </ci>
    </apply>
  </apply>
</math>
</formula>

```

In many elementary math books, you can find expressions like the previous. Because in our daily life we use the decimal numbering system, we can safely assume that $\langle \text{math} \rangle \langle \text{apply} \rangle \langle \text{eq}/\rangle \langle \text{ci} \rangle x \langle \text{ci} \rangle \langle \text{cn} \rangle 7 \langle \text{cn} \rangle \langle \text{apply} \rangle \langle \text{math} \rangle$. But, without explicitly mentioning this boundary condition, more solutions are correct.

```

<formula label="octal" sublabel="a">
  <math>
    <apply> <eq/>
      <cn> 10 </cn>
      <apply> <plus/>
        <cn> 3 </cn>
        <cn> 5 </cn>
      </apply>
    </apply>
  </math>
</formula>

```

In $\langle \text{textref label="octal"} \rangle \text{formula} \langle \text{textref} \rangle$ we see an at first sight wrong formula. But, if we tell you that octal numbers are used, your opinion may change instantly. A rather clean way out of this confusion is to extend the notation of numbers by explicitly mentioning the base.

```

<subformula label="octal base" sublabel="b">
  <math>
    <apply> <eq/>
      <cn type="integer" base="8"> 10 </cn>
      <apply> <plus/>
        <cn type="integer" base="8"> 3 </cn>
        <cn type="integer" base="8"> 5 </cn>
      </apply>
    </math>
  </subformula>

```

```

    </apply>
  </apply>
</math>
</subformula>

```

Of course, when a whole document is in octal notation, a proper introduction is better than annotated numbers as used in `<textref label="octal base">formula</textref>`.

Math that is part of the text flow is automatically handled as in line math. If needed you can encapsulate the code in an *imath* environment. Display math is recognized as such when it is a separate paragraph, but since this is more a T_EX feature than an XML one, you should encapsulate display math either in a *dmath* element or in a *formula* or *subformula* element.

Getting started

A comfortable way to get accustomed to MATHML is to make small documents of the following form:

```
\usemodule[mathml]
```

```
\starttext
```

```
\startXMLdata
```

```
<math>
```

```
  <apply> <cos/>
```

```
    <ci> x </ci>
```

```
  </apply>
```

```
</math>
```

```
\stopXMLdata
```

```
\stoptext
```

As you see, we can mix MATHML with normal \TeX code. A document like this is processed in the normal way using \TeX EXEC. If you also want to see the original code, you can say:

```
\usemodule[mathml]
```

```
\starttext
```

```
\startbuffer
```

```
<math>
```

```
  <apply> <cos/>
```

```
    <ci> x </ci>
```

```
  </apply>
```

```
</math>
```

```
\stopbuffer
```

```
\processXMLbuffer
```

```
\typebuffer
```

```
\stoptext
```

Like \TeX and \MetaPost code, buffers can contain MATHML code. The advantage of this method is that we only have to key in the data once. It also permits you to experiment with processing instructions.

```
\startbuffer[mm1]
<math>
  <apply> <log/>
    <logbase> <cn> 3.5 </cn> </logbase>
    <ci> x </ci>
  </apply>
</math>
\stopbuffer

\startbuffer[pi]
<?context-mathml-directive log location right?>
\stopbuffer

\processXMLbuffer[pi,mm1]

\startbuffer[pi]
<?context-mathml-directive log location left?>
\stopbuffer

\processXMLbuffer[pi,mm1]
```

If you like coding your documents in \TeX but want to experiment with MATHML, combining both languages in the way demonstrated here may be an option. When you provide enough structure in your \TeX code, converting a document to XML is then not that hard to do. Where coding directly in XML is kind of annoying, coding MATHML is less cumbersome, because you can structure your formulas pretty well, especially since the fragments are small so that proper indentation is possible.

A few examples

<> Derivatives

derivate 1

$$\frac{da}{dx} = 0$$

```
<math>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <ci> a </ci>
    </apply>
    <ci> 0 </ci>
  </apply>
</math>
```

derivate 2

$$\frac{dx}{dx} = 1$$

```
<math>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <ci> x </ci>
    </apply>
    <cn> 1 </cn>
  </apply>
</math>
```

derivate 3

$$\frac{d(au)}{dx} = a \frac{du}{dx}$$

```
<math>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <times/>
        <ci> a </ci>
        <ci> u </ci>
      </apply>
    </apply>
  </math>
```

```

<apply> <times/>
  <ci> a </ci>
  <apply> <diff/>
    <bvar> <ci> x </ci> </bvar>
    <ci> u </ci>
  </apply>
</apply>
</math>

```

derivate 4

$$\frac{d(u + v - w)}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx}$$

```

<math>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <plus/>
        <ci> u </ci>
        <ci> v </ci>
        <apply> <minus/>
          <ci> w </ci>
        </apply>
      </apply>
    </apply>
    <apply> <plus/>
      <apply> <diff/>
        <bvar> <ci> x </ci> </bvar>
        <ci> u </ci>
      </apply>
      <apply> <diff/>
        <bvar> <ci> x </ci> </bvar>
        <ci> v </ci>
      </apply>
      <apply> <diff/>
        <bvar> <ci> x </ci> </bvar>
        <ci> w </ci>
      </apply>
    </apply>
  </math>

```

A few examples

</apply>
</math>

derivate 5

$$\frac{d(uv)}{dx} = u \frac{du}{dx} + v \frac{dv}{dx}$$

<math>
 <apply> <eq/>
 <apply> <diff/>
 <bvar> <ci> x </ci> </bvar>
 <apply> <times/>
 <ci> u </ci>
 <ci> v </ci>
 </apply>
 </apply>
 <apply> <plus/>
 <apply> <times/>
 <ci> u </ci>
 <apply> <diff/>
 <bvar> <ci> x </ci>
 </bvar> <ci> u </ci>
 </apply>
 </apply>
 <apply> <times/>
 <ci> v </ci>
 <apply> <diff/>
 <bvar> <ci> x </ci>
 </bvar> <ci> v </ci>
 </apply>
 </apply>
 </apply>
 </math>

A few examples

derivate 6

$$\frac{d(uvw)}{dx} = uv \frac{dw}{dx} + vw \frac{du}{dx} + uw \frac{dv}{dx}$$

```

<math>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <times/>
        <ci> u </ci>
        <ci> v </ci>
        <ci> w </ci>
      </apply>
    </apply>
    <apply> <plus/>
      <apply> <times/>
        <ci> u </ci>
        <ci> v </ci>
      <apply> <diff/>
        <bvar> <ci> x </ci> </bvar>
        <ci> w </ci>
      </apply>
    </apply>
    <apply> <times/>
      <ci> v </ci>
      <ci> w </ci>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <ci> u </ci>
    </apply>
  </apply>
  <apply> <times/>
    <ci> u </ci>
    <ci> w </ci>
  <apply> <diff/>
    <bvar> <ci> x </ci> </bvar>
    <ci> v </ci>
  </apply>
</math>

```

A few examples

</apply>
</math>

derivate 7

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} = \frac{1}{v} \frac{du}{dx} - \frac{1}{u} \frac{dv}{dx}$$

```
<math>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <divide/>
        <ci> u </ci>
        <ci> v </ci>
      </apply>
    </apply>
    <apply> <divide/>
      <apply> <minus/>
        <apply> <times/>
          <ci> v </ci>
          <apply> <diff/>
            <bvar> <ci> x </ci> </bvar>
            <ci> u </ci>
          </apply>
        </apply>
      </apply>
      <apply> <times/>
        <ci> u </ci>
        <apply> <diff/>
          <bvar> <ci> x </ci> </bvar>
          <ci> v </ci>
        </apply>
      </apply>
    </apply>
    <apply> <power/>
      <ci> v </ci>
      <cn> 2 </cn>
    </apply>
  </apply>
  <apply> <minus/>
```

```

<apply> <times/>
  <apply> <divide/>
    <cn> 1 </cn>
    <ci> v </ci>
  </apply>
  <apply> <diff/>
    <bvar> <ci> x </ci> </bvar>
    <ci> u </ci>
  </apply>
</apply>
<apply> <times/>
  <apply> <divide/>
    <cn> 1 </cn>
    <ci> u </ci>
  </apply>
  <apply> <diff/>
    <bvar> <ci> x </ci> </bvar>
    <ci> v </ci>
  </apply>
</apply>
</apply>
</math>

```

derivate 8

$$\frac{d(u^n)}{dx} = n u^{n-1} \frac{du}{dx}$$

```

<math>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <power/>
        <ci> u </ci>
        <ci> n </ci>
      </apply>
    </apply>
    <apply> <times/>
      <ci> n </ci>
      <apply> <power/>

```

A few examples


```

    <ci> u </ci>
    <apply> <minus/>
      <ci> n </ci>
      <cn> 1 </cn>
    </apply>
  </apply>
  <apply> <diff/>
    <bvar> <ci> x </ci> </bvar>
    <ci> u </ci>
  </apply>
</apply>
</math>

```

derivate 9

$$\frac{d(\sqrt{u})}{dx} = \frac{1}{2\sqrt{u}} \frac{du}{dx}$$

```

<math>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <root/>
        <ci> u </ci>
      </apply>
    </apply>
    <apply> <times/>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <times/>
          <cn> 2 </cn>
          <apply> <root/>
            <ci> u </ci>
          </apply>
        </apply>
      </apply>
    </apply>
  </math>

```

A few examples

</apply>
 </apply>
 </math>

derivate 10
$$\frac{d\left(\frac{1}{u}\right)}{dx} = \left(-\frac{1}{u^2}\right) \frac{du}{dx}$$

<math>
 <apply> <eq/>
 <apply> <diff/>
 <bvar> <ci> x </ci> </bvar>
 <apply> <divide/>
 <cn> 1 </cn>
 <ci> u </ci>
 </apply>
 </apply>
 <apply> <times/>
 <apply> <minus/>
 <apply> <divide/>
 <cn> 1 </cn>
 <apply> <power/>
 <ci> u </ci>
 <cn> 2 </cn>
 </apply>
 </apply>
 <apply> <diff/>
 <bvar> <ci> x </ci> </bvar>
 <ci> u </ci>
 </apply>
 </apply>
 </math>

derivate 11

$$\frac{d}{dx} \frac{1}{u^n} = \left(-\frac{n}{u^{n+1}} \right) \frac{du}{dx}$$

```

<math>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <power/>
          <ci> u </ci>
          <cn> n </cn>
        </apply>
      </apply>
    </apply>
    <apply> <times/>
      <apply> <minus/>
        <apply> <divide/>
          <ci> n </ci>
          <apply> <power/>
            <ci> u </ci>
            <apply> <plus/>
              <ci> n </ci>
              <cn> 1 </cn>
            </apply>
          </apply>
        </apply>
      </apply>
    </apply>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <ci> u </ci>
    </apply>
  </apply>
</math>

```

A few examples

derivate 43

$$\frac{d(\sinh u)^{-1}}{dx} = \frac{d \log(u + \sqrt{u^2 + 1})}{dx} = \frac{1}{\sqrt{u^2 + 1}} \frac{du}{dx}$$

```

<math>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <inverse/>
        <apply> <sinh/>
          <ci> u </ci>
        </apply>
      </apply>
    </apply>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <log/>
        <apply> <plus/>
          <ci> u </ci>
          <apply> <root/>
            <apply> <plus/>
              <apply> <power/>
                <ci> u </ci>
                <cn> 2 </cn>
              </apply>
              <cn> 1 </cn>
            </apply>
          </apply>
        </apply>
      </apply>
      <apply> <times/>
        <apply> <divide/>
          <cn> 1 </cn>
          <apply> <root/>
            <apply> <plus/>
              <apply> <power/>
                <ci> u </ci>
                <cn> 2 </cn>
              </apply>
            </apply>
          </apply>
        </apply>
      </apply>
    </math>

```

A few examples

```

    </apply>
    <cn> 1 </cn>
  </apply>
</apply>
</apply>
<apply> <diff/>
  <bvar> <ci> x </ci> </bvar>
  <ci> u </ci>
</apply>
</apply>
</apply>
</math>

```

<> *Integrals*

integral 22
$$\int \frac{1}{x \sqrt{a^2 \pm x^2}} dx = -\frac{1}{a} \log \left(\frac{a + \sqrt{a^2 \pm x^2}}{x} \right)$$

```

<math>
  <apply> <eq/>
    <apply> <int/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <times/>
          <ci> x </ci>
          <apply> <root/>
            <apply> <fn> <ci> &plusminus; </ci> </fn>
            <apply> <power/>
              <ci> a </ci>
              <cn> 2 </cn>
            </apply>
            <apply> <power/>
              <ci> x </ci>
              <cn> 2 </cn>
            </apply>
          </apply>
        </apply>
      </apply>
    </apply>
  </math>

```

A few examples

```

    </apply>
  </apply>
</apply>
<apply> <minus/>
  <apply> <times/>
    <apply> <divide/>
      <cn> 1 </cn> <ci> a </ci>
    </apply>
  </apply>
  <apply> <log/>
    <apply> <divide/>
      <apply> <plus/>
        <ci> a </ci>
      </apply>
      <apply> <root/>
        <apply> <fn> <ci> &plusminus; </ci> </fn>
          <apply> <power/>
            <ci> a </ci>
            <cn> 2 </cn>
          </apply>
          <apply> <power/>
            <ci> x </ci>
            <cn> 2 </cn>
          </apply>
        </apply>
      </apply>
    </apply>
  </apply>
  <ci> x </ci>
</apply>
</apply>
</apply>
</math>

```

integral 61 $\int \frac{1}{a + b x^2} dx = \frac{1}{2\sqrt{-ab}} \log \left(\frac{a + x\sqrt{-ab}}{a - x\sqrt{-ab}} \right) \vee \frac{1}{\sqrt{-ab}} \tanh^{-1} \left(\frac{x\sqrt{-ab}}{a} \right)$

<math>

<apply> <eq/>

<apply> <int/>

<bvar> <ci> x </ci> </bvar>

<apply> <divide/>

<cn> 1 </cn>

<apply> <plus/>

<ci> a </ci>

<apply> <times/>

<ci> b </ci>

<apply> <power/>

<ci> x </ci>

<cn> 2 </cn>

</apply>

</apply>

</apply>

</apply>

</apply>

<apply> <or/>

<apply> <times/>

<apply> <divide/>

<cn> 1 </cn>

<apply> <times/>

<cn> 2 </cn>

<apply> <root/>

<apply> <minus/>

<apply> <times/>

<ci> a </ci>

<ci> b </ci>

</apply>

</apply>

</apply>

</apply>

</apply>

<apply> <log/>

```
<apply> <divide/>  
  <apply> <plus/>  
    <ci> a </ci>  
  <apply> <times/>  
    <ci> x </ci>  
  <apply> <root/>  
    <apply> <minus/>  
      <apply> <times/>  
        <ci> a </ci>  
        <ci> b </ci>  
      </apply>  
    </apply>  
  </apply>  
</apply>  
<apply> <minus/>  
  <ci> a </ci>  
  <apply> <times/>  
    <ci> x </ci>  
  <apply> <root/>  
    <apply> <minus/>  
      <apply> <times/>  
        <ci> a </ci>  
        <ci> b </ci>  
      </apply>  
    </apply>  
  </apply>  
</apply>  
</apply>  
</apply>  
<apply> <times/>  
  <apply> <divide/>  
    <cn> 1 </cn>  
  <apply> <root/>  
    <apply> <minus/>  
      <apply> <times/>
```



```

        <ci> a </ci>
        <ci> b </ci>
    </apply>
</apply>
</apply>
</apply>
<apply> <power/>
    <apply> <tanh/>
        <apply> <divide/>
            <apply> <times/>
                <ci> x </ci>
                <apply> <root/>
                    <apply> <minus/>
                        <apply> <times/>
                            <ci> a </ci>
                            <ci> b </ci>
                        </apply>
                    </apply>
                </apply>
            </apply>
        </apply>
    </apply>
</apply>
<ci> a </ci>
</apply>
</apply>
<apply> <minus/>
    <cn> 1 </cn>
</apply>
</apply>
</apply>
</math>

```

integral 380
$$\int \frac{1}{\cos(ax) 1 \pm \sin(ax)} dx = \mp \frac{1}{2a 1 \pm \sin(ax)} + \frac{1}{2a} \log \tan \left(\frac{\pi}{4} + \frac{ax}{2} \right)$$

```

<math>
    <apply> <eq/>
        <apply> <int/>
            <bvar> <ci> x </ci> </bvar>

```

A few examples

```

<apply> <divide/>
  <cn> 1 </cn>
  <apply> <times/>
    <apply> <cos/>
      <apply> <times/>
        <ci> a </ci>
        <ci> x </ci>
      </apply>
    </apply>
  </apply>
  <apply> <fn> <ci> &plusminus; </ci> </fn>
    <cn> 1 </cn>
    <apply> <sin/>
      <apply> <times/>
        <ci> a </ci>
        <ci> x </ci>
      </apply>
    </apply>
  </apply>
</apply>
<apply> <plus/>
  <apply> <fn> <ci> &minusplus; </ci> </fn>
    <apply> <divide/>
      <cn> 1 </cn>
      <apply> <times/>
        <cn> 2 </cn>
        <ci> a </ci>
      <apply> <fn> <ci> &plusminus; </ci> </fn>
        <cn> 1 </cn>
        <apply> <sin/>
          <apply> <times/>
            <ci> a </ci>
            <ci> x </ci>
          </apply>
        </apply>
      </apply>
    </apply>
  </apply>
</apply>

```

```

    </apply>
  </apply>
  <apply> <times/>
    <apply> <divide/>
      <cn> 1 </cn>
      <apply> <times/>
        <cn> 2 </cn>
        <ci> a </ci>
      </apply>
    </apply>
  </apply>
  <apply> <log/>
    <apply> <tan/>
      <apply> <plus/>
        <apply> <divide/>
          <ci> &pi; </ci>
          <cn> 4 </cn>
        </apply>
        <apply> <divide/>
          <apply> <times/>
            <ci> a </ci>
            <ci> x </ci>
          </apply>
          <cn> 2 </cn>
        </apply>
      </apply>
    </apply>
  </apply>
</math>

```

<> *Series*

serie 1

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$$

```

<math>
  <apply> <eq/>

```

A few examples

```

<apply> <plus/>
  <cn> 1 </cn>
  <apply> <minus/>
    <apply> <divide/>
      <cn> 1 </cn>
      <cn> 3 </cn>
    </apply>
  </apply>
<apply> <divide/>
  <cn> 1 </cn>
  <cn> 5 </cn>
</apply>
<apply> <minus/>
  <apply> <divide/>
    <cn> 1 </cn>
    <cn> 7 </cn>
  </apply>
</apply>
<ci> &cdots; </ci>
</apply>
<apply> <divide/>
  <ci> &pi; </ci>
  <cn> 4 </cn>
</apply>
</apply>
</math>

```

serie 2

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$$

```

<math>
  <apply> <eq/>
    <apply> <plus/>
      <cn> 1 </cn>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <power/>
          <cn> 2 </cn>
          <cn> 2 </cn>

```

A few examples

```

    </apply>
  </apply>
  <apply> <divide/>
    <cn> 1 </cn>
    <apply> <power/>
      <cn> 3 </cn>
      <cn> 2 </cn>
    </apply>
  </apply>
  <apply> <divide/>
    <cn> 1 </cn>
    <apply> <power/>
      <cn> 4 </cn>
      <cn> 2 </cn>
    </apply>
  </apply>
  <ci> &cdots; </ci>
</apply>
<apply> <divide/>
  <apply> <power/>
    <ci> &pi; </ci>
    <cn> 2 </cn>
  </apply>
  <cn> 6 </cn>
</apply>
</apply>
</math>

```

serie 3

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$$

```

<math>
  <apply> <eq/>
    <apply> <plus/>
      <cn> 1 </cn>
      <apply> <minus/>
        <apply> <divide/>
          <cn> 1 </cn>
          <apply> <power/>

```

A few examples

```

        <cn> 2 </cn>
        <cn> 2 </cn>
    </apply>
</apply>
</apply>
<apply> <divide/>
    <cn> 1 </cn>
    <apply> <power/>
        <cn> 3 </cn>
        <cn> 2 </cn>
    </apply>
</apply>
<apply> <minus/>
    <apply> <divide/>
        <cn> 1 </cn>
        <apply> <power/>
            <cn> 4 </cn>
            <cn> 2 </cn>
        </apply>
    </apply>
</apply>
<ci> &cdots; </ci>
</apply>
<apply> <divide/>
    <apply> <power/>
        <ci> &pi; </ci>
        <cn> 2 </cn>
    </apply>
    <cn> 12 </cn>
</apply>
</apply>
</math>

```

serie 1

$$\forall x \in \mathbb{R} \left| e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots \right.$$

```

<math>
  <apply> <forall/>
    <condition>
      <apply> <in/>
        <ci> x </ci>
        <ci> &reals; </ci>
      </apply>
    </condition>
  <apply> <eq/>
    <apply> <power/>
      <ci> &exponentiale; </ci>
      <ci> x </ci>
    </apply>
    <apply> <plus/>
      <cn> 1 </cn>
      <ci> x </ci>
      <apply> <divide/>
        <apply> <power/>
          <ci> x </ci>
          <cn> 2 </cn>
        </apply>
        <apply> <factorial/>
          <cn> 2 </cn>
        </apply>
      </apply>
      <apply> <divide/>
        <apply> <power/>
          <ci> x </ci>
          <cn> 3 </cn>
        </apply>
        <apply> <factorial/>
          <cn> 3 </cn>
        </apply>
      </apply>
    <ci> &cdots; </ci>
  </apply>

```

A few examples

```

    <apply> <divide/>
      <apply> <power/>
        <ci> x </ci>
        <ci> n </ci>
      </apply>
      <apply> <factorial/>
        <ci> n </ci>
      </apply>
    </apply>
    <ci> &cdots; </ci>
  </apply>
</apply>
</math>

```

serie 2 $\forall x \in \mathbb{R} \left| e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + (-1)^n \frac{x^n}{n!} \dots \right.$

```

<math>
  <apply> <forall/>
    <condition>
      <apply> <in/>
        <ci> x </ci>
        <ci> &reals; </ci>
      </apply>
    </condition>
    <apply> <eq/>
      <apply> <power/>
        <ci> &exponentiale; </ci>
        <apply> <minus/>
          <ci> x </ci>
        </apply>
      </apply>
      <apply> <plus/>
        <cn> 1 </cn>
        <apply> <minus/>
          <ci> x </ci>
        </apply>
      </apply>
    </apply>
  </math>

```



```

<apply> <divide/>
  <apply> <power/>
    <ci> x </ci>
    <cn> 2 </cn>
  </apply>
  <apply> <factorial/>
    <cn> 2 </cn>
  </apply>
</apply>
<apply> <minus/>
  <apply> <divide/>
    <apply> <power/>
      <ci> x </ci>
      <cn> 3 </cn>
    </apply>
    <apply> <factorial/>
      <cn> 3 </cn>
    </apply>
  </apply>
</apply>
<ci> &cdots; </ci>
<apply> <times/>
  <apply> <power/>
    <apply> <minus/>
      <cn> 1 </cn>
    </apply>
    <ci> n </ci>
  </apply>
<apply> <divide/>
  <apply> <power/>
    <ci> x </ci>
    <ci> n </ci>
  </apply>
  <apply> <factorial/>
    <ci> n </ci>
  </apply>
</apply>
<ci> &cdots; </ci>

```

```

    </apply>
  </apply>
</apply>
</math>

```

<> Logs

$\log 1$ $\forall a > 0 \wedge b > 0 \mid \log_g(ab) = \log_g a + \log_g b$

```

<math>
  <apply> <forall/>
    <condition>
      <apply> <and/>
        <apply> <gt/>
          <ci> a </ci>
          <cn> 0 </cn>
        </apply>
        <apply> <gt/>
          <ci> b </ci>
          <cn> 0 </cn>
        </apply>
      </apply>
    </condition>
  <apply> <eq/>
    <apply> <log/>
      <logbase> <ci> g </ci> </logbase>
      <apply> <times/>
        <ci> a </ci>
        <ci> b </ci>
      </apply>
    </apply>
    <apply> <plus/>
      <apply> <log/>
        <logbase> <ci> g </ci> </logbase>
        <ci> a </ci>
      </apply>
      <apply> <log/>

```

```

      <logbase> <ci> g </ci> </logbase>
      <ci> b </ci>
    </apply>
  </apply>
</apply>
</math>

```

log 2 $\forall a > 0 \wedge b > 0 \left| \log_g \left(\frac{a}{b} \right) = \log_g a - \log_g b \right.$

```

<math>
  <apply> <forall/>
    <condition>
      <apply> <and/>
        <apply> <gt/>
          <ci> a </ci>
          <cn> 0 </cn>
        </apply>
        <apply> <gt/>
          <ci> b </ci>
          <cn> 0 </cn>
        </apply>
      </apply>
    </condition>
    <apply> <eq/>
      <apply> <log/>
        <logbase> <ci> g </ci> </logbase>
        <apply> <divide/>
          <ci> a </ci>
          <ci> b </ci>
        </apply>
      </apply>
      <apply> <minus/>
        <apply> <log/>
          <logbase> <ci> g </ci> </logbase>
          <ci> a </ci>
        </apply>
        <apply> <log/>

```

```

      <logbase> <ci> g </ci> </logbase>
      <ci> b </ci>
    </apply>
  </apply>
</apply>
</math>

```

log 3 $\forall b \in \mathbb{R} \wedge a > 0 \mid \log_g^b a = b \log_g a$

```

<math>
  <apply> <forall/>
    <condition>
      <apply> <and/>
        <apply> <in/>
          <ci> b </ci>
          <ci> &reals; </ci>
        </apply>
        <apply> <gt/>
          <ci> a </ci>
          <cn> 0 </cn>
        </apply>
      </apply>
    </condition>
    <apply> <eq/>
      <apply> <power/>
        <apply> <log/>
          <logbase> <ci> g </ci> </logbase>
          <ci> a </ci>
        </apply>
        <ci> b </ci>
      </apply>
      <apply> <times/>
        <ci> b </ci>
        <apply> <log/>
          <logbase> <ci> g </ci> </logbase>
          <ci> a </ci>
        </apply>
      </apply>
    </eq/>
  </forall>
</math>

```

</apply>
 </apply>
 </apply>
 </math>

log 4

$$\forall a > 0 \left| \log_g a = \frac{\log_p a}{\log_p g} \right.$$

<math>
 <apply> <forall/>
 <condition>
 <apply> <and/>
 <apply> <gt/>
 <ci> a </ci>
 <cn> 0 </cn>
 </apply>
 </apply>
 </condition>
 <apply> <eq/>
 <apply> <log/>
 <logbase> <ci> g </ci> </logbase>
 <ci> a </ci>
 </apply>
 <apply> <divide/>
 <apply> <log/>
 <logbase> <ci> p </ci> </logbase>
 <ci> a </ci>
 </apply>
 <apply> <log/>
 <logbase> <ci> p </ci> </logbase>
 <ci> g </ci>
 </apply>
 </apply>
 </apply>
 </math>

<> *Goniometrics**gonio 1*

$$\sin (x + y) = \sin x \cos y + \cos x \sin y$$

```

<math>
  <apply> <eq/>
    <apply> <sin/>
      <apply> <plus/>
        <ci> x </ci>
        <ci> y </ci>
      </apply>
    </apply>
    <apply> <plus/>
      <apply> <times/>
        <apply> <sin/>
          <ci> x </ci>
        </apply>
        <apply> <cos/>
          <ci> y </ci>
        </apply>
      </apply>
      <apply> <times/>
        <apply> <cos/>
          <ci> x </ci>
        </apply>
        <apply> <sin/>
          <ci> y </ci>
        </apply>
      </apply>
    </apply>
  </apply>
</math>

```

A few examples

gonio 2

$$\sin (x-y)=\sin x \cos y-\cos x \sin y$$

```

<math>
  <apply> <eq/>
    <apply> <sin/>
      <apply> <minus/>
        <ci> x </ci>
        <ci> y </ci>
      </apply>
    </apply>
    <apply> <minus/>
      <apply> <times/>
        <apply> <sin/>
          <ci> x </ci>
        </apply>
        <apply> <cos/>
          <ci> y </ci>
        </apply>
      </apply>
      <apply> <times/>
        <apply> <cos/>
          <ci> x </ci>
        </apply>
        <apply> <sin/>
          <ci> y </ci>
        </apply>
      </apply>
    </apply>
  </math>

```

gonio 3

$$\cos (x+y)=\cos x \cos y-\sin x \sin y$$

```

<math>
  <apply> <eq/>
    <apply> <cos/>
      <apply> <plus/>
        <ci> x </ci>
        <ci> y </ci>
      </apply>
    </apply>
  </math>

```

A few examples

```

    </apply>
  </apply>
  <apply> <minus/>
    <apply> <times/>
      <apply> <cos/>
        <ci> x </ci>
      </apply>
      <apply> <cos/>
        <ci> y </ci>
      </apply>
    </apply>
    <apply> <times/>
      <apply> <sin/>
        <ci> x </ci>
      </apply>
      <apply> <sin/>
        <ci> y </ci>
      </apply>
    </apply>
  </apply>
</math>

```

gonio-4

$$\cos (x - y) = \cos x \cos y + \sin x \sin y$$

```

<math>
  <apply> <eq/>
    <apply> <cos/>
      <apply> <minus/>
        <ci> x </ci>
        <ci> y </ci>
      </apply>
    </apply>
    <apply> <plus/>
      <apply> <times/>
        <apply> <cos/>
          <ci> x </ci>
        </apply>

```

A few examples


```

    <apply> <cos/>
      <ci> y </ci>
    </apply>
  </apply>
  <apply> <times/>
    <apply> <sin/>
      <ci> x </ci>
    </apply>
    <apply> <sin/>
      <ci> y </ci>
    </apply>
  </apply>
</math>

```

gonio 5

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

```

<math>
  <apply> <eq/>
    <apply> <tan/>
      <apply> <plus/>
        <ci> x </ci>
        <ci> y </ci>
      </apply>
    </apply>
    <apply> <divide/>
      <apply> <plus/>
        <apply> <tan/>
          <ci> x </ci>
        </apply>
        <apply> <tan/>
          <ci> y </ci>
        </apply>
      </apply>
      <apply> <minus/>
        <cn> 1 </cn>
        <apply> <times/>

```

A few examples

```

    <apply> <tan/>
      <ci> x </ci>
    </apply>
    <apply> <tan/>
      <ci> y </ci>
    </apply>
  </apply>
</math>

```

gonio 6

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

```

<math>
  <apply> <eq/>
    <apply> <tan/>
      <apply> <minus/>
        <ci> x </ci>
        <ci> y </ci>
      </apply>
    </apply>
    <apply> <divide/>
      <apply> <minus/>
        <apply> <tan/>
          <ci> x </ci>
        </apply>
        <apply> <tan/>
          <ci> y </ci>
        </apply>
      </apply>
      <apply> <plus/>
        <cn> 1 </cn>
        <apply> <times/>
          <apply> <tan/>
            <ci> x </ci>
          </apply>
          <apply> <tan/>

```

```

        <ci> y </ci>
      </apply>
    </apply>
  </apply>
</math>

```

gonio 7 $\sin p + \sin q = 2 \sin \left(\frac{p+q}{2} \right) \cos \left(\frac{p-q}{2} \right)$

```

<math>
  <apply> <eq/>
    <apply> <plus/>
      <apply> <sin/>
        <ci> p </ci>
      </apply>
      <apply> <sin/>
        <ci> q </ci>
      </apply>
    </apply>
    <apply> <times/>
      <cn> 2 </cn>
      <apply> <sin/>
        <apply> <divide/>
          <apply> <plus/>
            <ci> p </ci>
            <ci> q </ci>
          </apply>
          <cn> 2 </cn>
        </apply>
      </apply>
      <apply> <cos/>
        <apply> <divide/>
          <apply> <minus/>
            <ci> p </ci>
            <ci> q </ci>
          </apply>
          <cn> 2 </cn>
        </apply>
      </apply>
    </apply>
  </math>

```

</apply>
 </apply>
 </apply>
 </apply>
 </math>

gonio 8 $\sin p - \sin q = 2 \cos \left(\frac{p+q}{2} \right) \sin \left(\frac{p-q}{2} \right)$

<math>
 <apply> <eq/>
 <apply> <minus/>
 <apply> <sin/>
 <ci> p </ci>
 </apply>
 <apply> <sin/>
 <ci> q </ci>
 </apply>
 </apply>
 <apply> <times/>
 <cn> 2 </cn>
 <apply> <cos/>
 <apply> <divide/>
 <apply> <plus/>
 <ci> p </ci>
 <ci> q </ci>
 </apply>
 <cn> 2 </cn>
 </apply>
 </apply>
 <apply> <sin/>
 <apply> <divide/>
 <apply> <minus/>
 <ci> p </ci>
 <ci> q </ci>
 </apply>
 <cn> 2 </cn>
 </apply>
 </apply>
 </math>

</apply>
 </apply>
 </math>

gonio-9 $\cos p + \cos q = 2 \cos \left(\frac{p+q}{2} \right) \cos \left(\frac{p-q}{2} \right)$

<math>
 <apply> <eq/>
 <apply> <plus/>
 <apply> <cos/>
 <ci> p </ci>
 </apply>
 <apply> <cos/>
 <ci> q </ci>
 </apply>
 </apply>
 <apply> <times/>
 <cn> 2 </cn>
 <apply> <cos/>
 <apply> <divide/>
 <apply> <plus/>
 <ci> p </ci>
 <ci> q </ci>
 </apply>
 <cn> 2 </cn>
 </apply>
 </apply>
 <apply> <cos/>
 <apply> <divide/>
 <apply> <minus/>
 <ci> p </ci>
 <ci> q </ci>
 </apply>
 <cn> 2 </cn>
 </apply>
 </apply>
 </apply>
 </apply>
 </math>

</math>

gonio 10

$$\cos p - \cos q = -2 \sin \left(\frac{p+q}{2} \right) \sin \left(\frac{p-q}{2} \right)$$

<math>

<apply> <eq/>

<apply> <minus/>

<apply> <cos/>

<ci> p </ci>

</apply>

<apply> <cos/>

<ci> q </ci>

</apply>

</apply>

<apply> <minus/>

<apply> <times/>

<cn> 2 </cn>

<apply> <sin/>

<apply> <divide/>

<apply> <plus/>

<ci> p </ci>

<ci> q </ci>

</apply>

<cn> 2 </cn>

</apply>

</apply>

<apply> <sin/>

<apply> <divide/>

<apply> <minus/>

<ci> p </ci>

<ci> q </ci>

</apply>

<cn> 2 </cn>

</apply>

</apply>

</apply>

</apply>

</apply>

</math>

gonio-11

$$2 \sin \alpha \cos \beta = \sin (\alpha + \beta) + \sin (\alpha - \beta)$$

<math>

<apply> <eq/>

<apply> <times/>

<cn> 2 </cn>

<apply> <sin/>

<ci> α </ci>

</apply>

<apply> <cos/>

<ci> β </ci>

</apply>

</apply>

<apply> <plus/>

<apply> <sin/>

<apply> <plus/>

<ci> α </ci>

<ci> β </ci>

</apply>

</apply>

<apply> <sin/>

<apply> <minus/>

<ci> α </ci>

<ci> β </ci>

</apply>

</apply>

</apply>

</apply>

</math>

gonio-12

$$2 \cos \alpha \sin \beta = \sin (\alpha + \beta) - \sin (\alpha - \beta)$$

<math>

<apply> <eq/>

<apply> <times/>

<cn> 2 </cn>

<apply> <cos/>

A few examples

```

    <ci> &alpha; </ci>
  </apply>
  <apply> <sin/>
    <ci> &beta; </ci>
  </apply>
</apply>
<apply> <minus/>
  <apply> <sin/>
    <apply> <plus/>
      <ci> &alpha; </ci>
      <ci> &beta; </ci>
    </apply>
  </apply>
<apply> <sin/>
  <apply> <minus/>
    <ci> &alpha; </ci>
    <ci> &beta; </ci>
  </apply>
</apply>
</apply>
</math>

```

gonio-13

$$2 \cos \alpha \cos \beta = \cos (\alpha + \beta) + \cos (\alpha - \beta)$$

```

<math>
  <apply> <eq/>
    <apply> <times/>
      <cn> 2 </cn>
      <apply> <cos/>
        <ci> &alpha; </ci>
      </apply>
      <apply> <cos/>
        <ci> &beta; </ci>
      </apply>
    </apply>
    <apply> <plus/>
      <apply> <cos/>

```



```

    <apply> <plus/>
      <ci> &alpha; </ci>
      <ci> &beta; </ci>
    </apply>
  </apply>
  <apply> <cos/>
    <apply> <minus/>
      <ci> &alpha; </ci>
      <ci> &beta; </ci>
    </apply>
  </apply>
</math>

```

gonio 14 $-2 \sin \alpha \cos \beta = \sin (\alpha + \beta) - \sin (\alpha - \beta)$

```

<math>
  <apply> <eq/>
    <apply> <minus/>
      <apply> <times/>
        <cn> 2 </cn>
        <apply> <sin/>
          <ci> &alpha; </ci>
        </apply>
        <apply> <cos/>
          <ci> &beta; </ci>
        </apply>
      </apply>
    </apply>
    <apply> <minus/>
      <apply> <sin/>
        <apply> <plus/>
          <ci> &alpha; </ci>
          <ci> &beta; </ci>
        </apply>
      </apply>
    </apply>
    <apply> <sin/>

```

```

      <apply> <minus/>
        <ci> &alpha; </ci>
        <ci> &beta; </ci>
      </apply>
    </apply>
  </apply>
</math>

```

gonio-15

$$\forall \triangle ABC \left| \frac{a}{\sin \alpha} = \frac{a}{\sin \beta} = \frac{a}{\sin \gamma} \right.$$

```

<math>
  <apply> <forall/>
    <condition>
      <mrow>
        <mi> &triangle; </mi>
        <mi> A </mi>
        <mi> B </mi>
        <mi> C </mi>
      </mrow>
    </condition>
    <apply> <eq/>
      <apply> <divide/>
        <ci> a </ci>
        <apply> <sin/>
          <ci> &alpha; </ci>
        </apply>
      </apply>
      <apply> <divide/>
        <ci> a </ci>
        <apply> <sin/>
          <ci> &beta; </ci>
        </apply>
      </apply>
      <apply> <divide/>
        <ci> a </ci>
        <apply> <sin/>

```

```

      <ci> &gamma; </ci>
    </apply>
  </apply>
</math>

```

gonio 16

$$\forall \triangle ABC \left| \begin{array}{l} a^2 = b^2 + c^2 - 2bc \cos \alpha \\ b^2 = a^2 + c^2 - 2ac \cos \beta \\ c^2 = a^2 + b^2 - 2ab \cos \gamma \end{array} \right.$$

```

<math>
  <apply> <forall/>
    <condition>
      <mrow>
        <mi> &triangle; </mi>
        <mi> A </mi>
        <mi> B </mi>
        <mi> C </mi>
      </mrow>
    </condition>
  <apply> <eq/>
    <apply> <power/>
      <ci> a </ci>
      <cn> 2 </cn>
    </apply>
    <apply> <plus/>
      <apply> <power/>
        <ci> b </ci>
        <cn> 2 </cn>
      </apply>
      <apply> <power/>
        <ci> c </ci>
        <cn> 2 </cn>
      </apply>
      <apply> <minus/>
        <apply> <times/>
          <cn> 2 </cn>
          <ci> b </ci>

```

```

    <ci> c </ci>
    <apply> <cos/>
      <ci> &alpha; </ci>
    </apply>
  </apply>
</apply>
<apply> <eq/>
  <apply> <power/>
    <ci> b </ci>
    <cn> 2 </cn>
  </apply>
  <apply> <plus/>
    <apply> <power/>
      <ci> a </ci>
      <cn> 2 </cn>
    </apply>
    <apply> <power/>
      <ci> c </ci>
      <cn> 2 </cn>
    </apply>
    <apply> <minus/>
      <apply> <times/>
        <cn> 2 </cn>
        <ci> a </ci>
        <ci> c </ci>
        <apply> <cos/>
          <ci> &beta; </ci>
        </apply>
      </apply>
    </apply>
  </apply>
</apply> <eq/>
  <apply> <power/>
    <ci> c </ci>
    <cn> 2 </cn>

```

```

</apply>
<apply> <plus/>
  <apply> <power/>
    <ci> a </ci>
    <cn> 2 </cn>
  </apply>
  <apply> <power/>
    <ci> b </ci>
    <cn> 2 </cn>
  </apply>
  <apply> <minus/>
    <apply> <times/>
      <cn> 2 </cn>
      <ci> a </ci>
      <ci> b </ci>
    <apply> <cos/>
      <ci> &gamma; </ci>
    </apply>
  </apply>
</apply>
</math>

```

<> *Statistics*

statistic 1

$$\bar{x} = \frac{1}{n} \sum x_i$$

```

<math>
  <apply> <eq/>
    <apply> <mean/>
      <ci> x </ci>
    </apply>
    <apply> <times/>
      <apply> <divide/>
        <cn> 1 </cn>
        <ci> n </ci>
      </apply>
    </apply>
  </math>

```

A few examples

```

    </apply>
    <apply> <sum/>
      <ci> <msub> <mi> x </mi> <mi> i </mi> </msub> </ci>
    </apply>
  </apply>
</math>

```

statistic 2

$$\sigma(x) \approx \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}$$

```

<math>
  <apply> <approx/>
    <apply> <sdev/>
      <ci> x </ci>
    </apply>
    <apply> <root/>
      <apply> <divide/>
        <apply> <sum/>
          <apply> <power/>
            <apply> <minus/>
              <ci> <msub> <mi> x </mi> <mi> i </mi> </msub> </ci>
              <apply> <mean/>
                <ci> x </ci>
              </apply>
            </apply>
          <cn> 2 </cn>
        </apply>
      </apply>
      <apply> <minus/>
        <ci> n </ci>
        <cn> 1 </cn>
      </apply>
    </apply>
  </apply>
</math>

```

A few examples

statistic 3

$$\sigma(x)^2 \approx \overline{(x_i - \bar{x})^2} = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

```

<math>
  <apply> <approx/>
    <apply> <variance/>
      <ci> x </ci>
    </apply>
  <apply> <eq/>
    <apply> <mean/>
      <apply> <power/>
        <apply> <minus/>
          <ci> <msub> <mi> x </mi> <mi> i </mi> </msub> </ci>
          <apply> <mean/>
            <ci> x </ci>
          </apply>
        </apply>
      <cn> 2 </cn>
    </apply>
  </apply>
  <apply> <times/>
    <apply> <divide/>
      <cn> 1 </cn>
      <apply> <minus/>
        <ci> n </ci>
        <cn> 1 </cn>
      </apply>
    </apply>
  <apply> <sum/>
    <apply> <power/>
      <apply> <minus/>
        <ci> <msub> <mi> x </mi> <mi> i </mi> </msub> </ci>
        <apply> <mean/>
          <ci> x </ci>
        </apply>
      </apply>
    <cn> 2 </cn>
  </apply>
</apply>

```

A few examples

</apply>
 </apply>
 </apply>
 </math>

<> *Matrices*

matrix 1

$$\begin{vmatrix} \sin \alpha & \cos \alpha \\ \sin \beta & \cos \beta \end{vmatrix} = \sin (\alpha - \beta)$$

<math>
 <apply> <eq/>
 <apply> <determinant/>
 <matrix>
 <matrixrow>
 <apply> <sin/> <ci> α </ci> </apply>
 <apply> <cos/> <ci> α </ci> </apply>
 </matrixrow>
 <matrixrow>
 <apply> <sin/> <ci> β </ci> </apply>
 <apply> <cos/> <ci> β </ci> </apply>
 </matrixrow>
 </matrix>
 </apply>
 <apply> <sin/>
 <apply> <minus/>
 <ci> α </ci>
 <ci> β </ci>
 </apply>
 </apply>
 </apply>
 </math>

matrix 2

$$|I| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

<math>
 <apply> <eq/>
 <apply> <determinant/>

A few examples


```

    <ci> I </ci>
  </apply>
<apply> <determinant/>
  <matrix>
    <matrixrow> <cn> 1 </cn> <cn> 0 </cn> </matrixrow>
    <matrixrow> <cn> 0 </cn> <cn> 1 </cn> </matrixrow>
  </matrix>
</apply>
<cn> 1 </cn>
</apply>
</math>

```










































Primary entities

This chapter is one big table of entities. This list is derived from the lists posted on the www by Patrick Ion from the American Mathematics Society.

<> *Arrow relations*

	angzarr	E248	angle with down zig-zag arrow
	cirmid	E250	circle, mid below
	cudarrl	E23E	left, curved, down arrow
	cudarr	E400	right, curved, down arrow
	cularr	21B6	left curved arrow
	cularrp	E24A	curved left arrow with plus
	curarr	21B7	right curved arrow
	curarrm	E249	curved right arrow with minus
	Darr	21A1	down two-headed arrow
	dArr	21D3	down double arrow
	ddarr	21CA	two down arrows
	DDottrahd	E238	right arrow with dotted stem
	dfisht	E24C	down fish tail
	dHar	E227	down harpoon-left, down harpoon-right
	dharl	21C3	down harpoon-left
	dharr	21C2	down harpoon-rt
	duarr	E216	down arrow, up arrow
	duhar	E217	down harp, up harp
	dzigrarr	21DD	right long zig-zag arrow
	erarr	E236	equal, right arrow below
	hArr	21D4	left and right double arrow
	harr	2194	left and right arrow
	harrcir	E240	left and right arrow with a circle
	harrw	21AD	left and right arr-wavy
	hoarr	E243	horizontal open arrow
	imof	22B7	image of
	lAarr	21DA	left triple arrow
	Larr	219E	twoheadleftarrow
	larrbfs	E220	left arrow-bar, filled square
	larrfs	E222	left arrow, filled square
	larrhk	21A9	left arrow-hooked
	larrlp	21AB	left arrow-looped
	larrpl	E23F	left arrow, plus
	larrrsim	E24E	left arrow, similar

?	larrtl	21A2	left arrow-tailed
?	lAtail	E23D	left double arrow-tail
?	latail	E23C	left arrow-tail
?	lBarr	E206	left doubly broken arrow
?	lbarr	E402	left broken arrow
?	ldca	E21A	left down curved arrow
?	ldrdhar	E22C	left harpoon-down over right harpoon-down
?	ldrushar	E228	left-down-right-up harpoon
?	ldsh	21B2	left down angled arrow
?	lfisht	E214	left fish tail
?	lHar	E225	left harpoon-up over left harpoon-down
←	lhard	21BD	left harpoon-down
←	lharu	21BC	left harpoon-up
?	lharul	E22E	left harpoon-up over long dash
?	llarr	21C7	two left arrows
?	llhard	E231	left harpoon-down below long dash
?	loarr	E242	left open arrow
?	lrarr	21C6	left arr over right arr
?	lrhar	21CB	left harp over r
?	lrhard	E22F	right harpoon-down below long dash
↵	lsh	21B0	Lsh
?	lurdshar	E229	left-up-right-down harpoon
?	luruhar	E22B	left harpoon-up over right harpoon-up
?	Map	E212	twoheaded mapsto
↦	map	21A6	mapsto
?	midcir	E20F	mid, circle below
?	mumap	22B8	multimap
?	nearhk	E20D	NE arrow-hooked
?	neArr	21D7	NE pointing double arrow
↗	nearr	2197	NE pointing arrow
?	nesear	E20E	NE & SE arrows
?	nhArr	21CE	not left and right double arr
?	nharr	21AE	not left and right arrow
?	nłArr	21CD	not implied by
?	nłarr	219A	not left arrow
?	nrArr	21CF	not implies
?	nrarr	219B	not right arrow
?	nrarrc	E21D	not right arrow-curved
?	nrarrw	E21B	not right arrow-wavy
?	nvhArr*	21CE	not, vert, left and right double arrow

	nvlArr	21CD	not, vert, left double arrow
	nvrArr	21CF	not, vert, right double arrow
	nwarhk	E20C	NW arrow-hooked
	nwArr	21D6	NW pointing double arrow
	nwarr	2196	NW pointing arrow
	nwnear	E211	NW & NE arrows
	olarr	21BA	left arr in circle
	orarr	21BB	right arr in circle
	origof	22B6	original of
	rAarr	21DB	right triple arrow
	Rarr	21A0	twoheadrightarrow
	rarrap	E235	approximate, right arrow above
	rarrbfs	E221	right arrow-bar, filled square
	rarrc	E21C	right arrow-curved
	rarrfs	E223	right arrow, filled square
	rarrhk	21AA	right arrow-hooked
	rarrlp	21AC	right arrow-looped
	rarrpl	E21E	right arrow, plus
	rarrsim	E24D	right arrow, similar
	Rarrtl	E239	right two-headed arrow with tail
	rarrtl	21A3	right arrow-tailed
	rarrw	219D	right arrow-wavy
	rAtail	E23B	right double arrow-tail
	ratail	21A3	right arrow-tail
	RBarr	E209	twoheaded right broken arrow
	rBarr	E207	right doubly broken arrow
	rbarr	E405	right broken arrow
	rdca	E219	right down curved arrow
	rdldhar	E22D	right harpoon-down over left harpoon-down
	rdsh	21B3	right down angled arrow
	rfisht	E215	right fish tail
	rHar	E224	right harpoon-up over right harpoon-down
	rhard	21C1	right harpoon-down
	rharu	21C0	right harpoon-up
	rharul	E230	right harpoon-up over long dash
	rlarr	21C4	right arr over left arr
	rlhar	21CC	right harp over l
	roarr	E241	right open arrow
	rrarr	21C9	two right arrows
	rsh	21B1	Rsh

?	ru \downarrow har	E22A	right harpoon-up over left harpoon-up
?	searhk	E20B	SE arrow-hooked
?	seArr	21D8	SE pointing double arrow
↘	searr	2198	SE pointing arrow
?	seswar	E406	SE & SW arrows
?	simrarr	E234	similar, right arrow below
?	slarr	E233	short left arrow
?	srarr	E232	short right arrow
?	swarhk	E20A	SW arrow-hooked
?	swArr	21D9	SW pointing double arrow
↙	swarr	2199	SW pointing arrow
?	swnwar	E210	SW & NW arrows
?	Uarr	219F	up two-headed arrow
↑	uArr	21D1	up double arrow
?	Uarroci \circ r	E237	up two-headed arrow above circle
?	udarr	21C5	up arrow, down arrow
?	udhar	E218	up harp, down harp
?	ufisht	E24B	up fish tail
?	uHar	E226	up harpoon-left, up harpoon-right
?	uhar \downarrow	21BF	up harpoon-left
?	uharr	21BE	up harp-r
?	uuarr	21C8	two up arrows
↕	vArr	21D5	up & down double arrow
↑	varr	2195	up & down arrow
?	xhArr	E202	long left and right double arr
?	xharr	E203	long left and right arr
?	x \downarrow Arr	E200	long left double arrow
?	x \downarrow arr	E201	long left arrow
?	xmap	E208	longmapsto
?	xrArr	E204	long right double arr
?	xrarr	E205	long right arrow
?	zigrarr	E244	right zig-zag arrow

<> *Binary operators*

?	ac	E207	most positive
?	acE	E290	most positive, two lines below
II	amalg	E251	amalgamation or coproduct
?	barvee	22BD	bar, vee
?	Barwed	2306	logical and, double bar above

Primary entities

?	barwed	22BC	logical and, bar above
?	bsolb	E280	reverse solidus in square
?	Cap	22D2	double intersection
?	capand	E281	intersection, and
?	capbrcup	E271	intersection, bar, union
?	capcap	E273	intersection, intersection, joined
?	capcup	E26F	intersection above union
?	capdot	E261	intersection, with dot
?	caps	E275	intersection, serifs
?	ccaps	E279	closed intersection, serifs
?	ccups	E278	closed union, serifs
?	ccupssm	E27A	closed union, serifs, smash product
⊔	coprod	2210	coproduct operator
?	Cup	22D3	double union
?	cupbrcap	E270	union, bar, intersection
?	cupcap	E26E	union above intersection
?	cupcup	E272	union, union, joined
?	cupdot	228D	union, with dot
?	cupor	E282	union, or
?	cups	E274	union, serifs
?	cuvee	22CE	curly logical or
?	cuwed	22CF	curly logical and
‡	Dagger	2021	double dagger relation
†	dagger	2020	dagger relation
?	diam	22C4	open diamond
?	divonx	22C7	division on times
?	eplus	E268	equal, plus
?	hercon	22B9	hermitian conjugate matrix
⌊	intcal	22BA	intercal
?	iprod	E259	interior product
?	loplus	E25C	plus sign in left half circle
?	lotimes	E25E	multiply sign in left half circle
?	lthree	22CB	leftthreetimes
?	ltimes	22C9	times sign, left closed
*	midast	2217	centered asterisk
?	minusb	229F	minus sign in box
?	minUSD	2238	minus sign, dot above
?	minusdu	E25B	minus sign, dot below
?	ncap	E284	bar, intersection
?	ncup	E283	bar, union

$\textcircled{?}$	oast	229B	asterisk in circle
$\textcircled{?}$	ocir	229A	small circle in circle
$\textcircled{?}$	odash	229D	hyphen in circle
$\textcircled{?}$	odiv	E285	divide in circle
$\textcircled{\circ}$	odot	2299	middle dot in circle
$\textcircled{?}$	odsold	E286	dot, solidus, dot in circle
$\textcircled{?}$	ofcir	E287	filled circle in circle
$\textcircled{?}$	ogt	E289	greater-than in circle
$\textcircled{?}$	ohbar	E260	circle with horizontal bar
$\textcircled{?}$	olcir	E409	large circle in circle
$\textcircled{?}$	olt	E288	less-than in circle
$\textcircled{?}$	omid	E40A	vertical bar in circle
\ominus	ominus	2296	minus sign in circle
$\textcircled{?}$	opar	E28A	parallel in circle
$\textcircled{?}$	operp	E28B	perpendicular in circle
\oplus	oplus	2295	plus sign in circle
$\textcircled{?}$	osol	2298	solidus in circle
$\textcircled{?}$	Otimes	E28C	multiply sign in double circle
\otimes	otimes	2297	multiply sign in circle
$\textcircled{?}$	otimesas	E28D	multiply sign in circle, circumflex accent
$\textcircled{?}$	ovbar	E40B	circle with vertical bar
$\textcircled{?}$	plusacir	E26A	plus, circumflex accent above
$\textcircled{?}$	plusb	229E	plus sign in box
$\textcircled{?}$	pluscir	E266	plus, small circle above
$\textcircled{?}$	plusdo	2214	plus sign, dot above
$\textcircled{?}$	plusdu	E25A	plus sign, dot below
$\textcircled{?}$	pluse	E267	plus, equals
$\textcircled{?}$	plussim	E26C	plus, similar below
$\textcircled{?}$	plustwo	E269	plus, two; Nim-addition
$\textcircled{?}$	race	E40C	reverse most positive, line below
$\textcircled{?}$	roplus	E25D	plus sign in right half circle
$\textcircled{?}$	rotimes	E40D	multiply sign in right half circle
$\textcircled{?}$	rthree	22CC	rightthreetimes
$\textcircled{?}$	rtimes	22CA	times sign, right closed
\cdot	sdot	22C5	small middle dot
$\textcircled{?}$	sdotb	22A1	small dot in box
$\textcircled{?}$	setmn	2216	reverse solidus
$\textcircled{?}$	simplus	E26B	plus, similar above
$\textcircled{?}$	smashp	E264	smash product
$\textcircled{?}$	solb	E27F	solidus in square

\sqcap	sqcap	2293	square intersection
\squarecap	sqcaps	E277	square intersection, serifs
\sqcup	sqcup	2294	square union
\squarecup	sqcups	E276	square union, serifs
\smile	ssetmn	E844	sm reverse solidus
\star	sstarf	22C6	small star, filled, low
\subset	subdot	E262	subset, with dot
\sum	sum	2211	summation operator
\supset	supdot	E263	superset, with dot
\boxtimes	timesb	22A0	multiply sign in box
\boxtimes	timesbar	E28E	multiply sign, bar below
\cdot	timesd	E26D	times, dot
\triangle	tridot	25EC	dot in triangle
\triangleleft	triminus	E27C	minus in triangle
\triangleplus	triplus	E27B	plus in triangle
\triangle	trisb	E27E	triangle, serifs at bottom
\triangle	tritime	E27D	multiply in triangle
\uplus	uplus	228E	plus sign in union
\vee	veebar	22BB	logical or, bar below
\wedge	wedbar	E265	wedge, bar below
\wr	wreath	2240	wreath product
\cap	xcap	22C2	intersection operator
\bigcirc	xcirc	25CB	large circle
\bigcup	xcup	22C3	union operator
∇	xdtri	25BD	big down triangle, open
\odot	xodot	2299	circle dot operator
\oplus	xoplus	2295	circle plus operator
\otimes	xotime	2297	circle times operator
\sqcup	xsqcup	2294	square union operator
\uplus	xuplus	228E	biguplus
\triangle	xutri	25B3	big up triangle, open
\vee	xvee	22C1	logical and operator
\wedge	xwedge	22C0	logical or operator

<> Delimiters

\lrcorner	dllcorn	231E	lower left corner
\llcorner	drcorn	231F	lower right corner
$\left($	gtlPar	E296	double left parenthesis, greater
\angle	langd	E297	left angle, dot
$\left[$	lbrke	E299	left bracket, equal

?	lbrksld	E29D	left bracket, solidus bottom corner
?	lbrkslu	E29B	left bracket, solidus top corner
[lceil	2308	left ceiling
⌊	lfloor	230A	left floor
˘	lmoust	E294	left moustache
?	lparlt	E292	left parenthesis, lt
?	ltrPar	E295	double right parenthesis, less
?	rangd	E298	right angle, dot
?	rbrke	E29A	right bracket, equal
?	rbrksld	E29C	right bracket, solidus bottom corner
?	rbrkslu	E29E	right bracket, solidus top corner
⌋	rceil	2309	right ceiling
⌋	rfloor	230B	right floor
˘	rmoust	E293	right moustache
?	rpargt	E291	right paren, gt
?	ulcorn	231C	upper left corner
?	urcorn	231D	upper right corner

<> *General Technical*

?	acd	E3A6	ac current
ℵ	aleph	2135	aleph, Hebrew
?	And	E374	double logical and
^	and	2227	logical and
?	andand	E36E	two logical and
?	andd	E394	and, horizontal dash
?	andslope	E50A	sloping large and
?	andv	E391	and with middle stem
?	angrt	221F	right (90 degree) angle
?	angsph	2222	angle-spherical
Å	angst	212B	Angstrom capital A, ring
≈	ap	2248	approximate
?	apacir	E38C	approximate, circumflex accent
?	awconint	2233	contour integral, anti-clockwise
?	awint	E39B	anti clock-wise integration
?	becaus	2235	because
?	bernou	212C	Bernoulli function (script capital B)
?	bne	E388	reverse not equal
?	bnequiv	E387	reverse not equivalent
?	bNot	E3AD	reverse not with two horizontal strokes

\neg	bnot	2310	reverse not
\perp	bottom	22A5	bottom
\cap	cap	2229	intersection
\oint	Cconint	2230	triple contour integral operator
\oint	cirfnint	E395	circulation function
\circ	compfn	2218	composite function (small circle)
\cong	cong	2245	congruent with
\oint	Conint	222F	double contour integral operator
\oint	conint	222E	contour integral operator
\cdots	ctdot	22EF	three dots, centered
\cup	cup	222A	union or logical sum
\oint	cwconint	2232	contour integral, clockwise
\oint	cwint	2231	clockwise integral
\oint	cylcty	232D	cylindricity
\oint	disin	E3A0	set membership, long horizontal stroke
$\ddot{}$	Dot	0308	dieresis or umlaut mark
$\ddot{}$	DotDot	20DC	four dots above
$\overline{}$	dso1	E3A9	solidus, bar above
$\dot{}$	dtidot	22F1	three dots, descending
\sphericalangle	dwangle	E3AA	large downward pointing angle
\oint	elinters	E3A7	electrical intersection
\parallel	epar	22D5	parallel, equal; equal or parallel
\parallel	eparsl	E384	parallel, slanted, equal; homothetically co
\equiv	equiv	2261	identical with
\parallel	eqvparsl	E386	equivalent, equal; congruent and parallel
\exists	exist	2203	at least one exists
\flat	fltns	E381	flatness
\forall	forall	2200	for all
\oint	fpartint	E396	finite part integral
\geq	ge	2265	greater-than-or-equal
\mathcal{H}	hamilt	210B	Hamiltonian (script capital H)
iff	iff	E365	if and only if
∞	iinfin	E372	infinity sign, incomplete
imped	imped	E50B	impedance
∞	infin	221E	infinity
∞	infintie	E50C	tie, infinity
\oint	Int	222C	double integral operator
\int	int	222B	integral operator
\int	intlark	E39A	integral, left arrow with hook
\in	isin	220A	set membership
$\dot{\in}$	isindot	E39C	set membership, dot above

$\mathbb{?}$	isinE	E39E	set membership, two horizontal strokes
$\mathbb{?}$	isins	E3A4	set membership, vertical bar on horizontal
$\mathbb{?}$	isinsv	E3A2	large set membership, vertical bar on horiz
$\mathbb{?}$	isinv	2208	set membership, variant
$\mathbb{?}$	lagran	2112	Lagrangian (script capital L)
$\mathbb{?}$	Lang	300A	left angle bracket, double
\langle	lang	3008	left angle bracket
\Leftarrow	lArr	21D0	is implied by
$\mathbb{?}$	lbrbrk	3014	left broken bracket
\leq	le	2264	less-than-or-equal
$\mathbb{?}$	loang	3018	left open angular bracket
$\mathbb{?}$	lobrk	301A	left open bracket
$\mathbb{?}$	lopar	E379	left open parenthesis
$\mathbb{?}$	lowast	2217	low asterisk
$-$	minus	2212	minus sign
\mp	mnplus	2213	minus-or-plus sign
∇	nabla	2207	del, Hamilton operator
\neq	ne	2260	not equal
$\mathbb{?}$	nedot	E38A	not equal, dot
$\mathbb{?}$	nhpar	E38D	not, horizontal, parallel
$\mathbb{?}$	ni	220D	contains
$\mathbb{?}$	nis	E3A5	contains, vertical bar on horizontal stroke
$\mathbb{?}$	nisd	E3A1	contains, long horizontal stroke
\ni	niv	220B	contains, variant
$\mathbb{?}$	Not	E3AC	not with two horizontal strokes
\notin	notin	2209	negated set membership
$\mathbb{?}$	notindot	E39D	negated set membership, dot above
$\mathbb{?}$	notinE	E50D	negated set membership, two horizontal strokes
$\mathbb{?}$	notinva	E370	negated set membership, variant
$\mathbb{?}$	notinvb	E37B	negated set membership, variant
$\mathbb{?}$	notinvc	E37C	negated set membership, variant
$\mathbb{?}$	notni	220C	negated contains
$\mathbb{?}$	notniva	220C	negated contains, variant
$\mathbb{?}$	notnivb	E37D	contains, variant
$\mathbb{?}$	notnivc	E37E	contains, variant
$\mathbb{?}$	nparsl	E389	not parallel, slanted
$\mathbb{?}$	npart	E390	not partial differential
$\mathbb{?}$	npolint	E399	line integration, not including the pole
$\mathbb{?}$	nvinfin	E38E	not, vert, infinity
$\mathbb{?}$	olcross	E3A8	circle, cross
$\mathbb{?}$	Or	E375	double logical or

∨	or	2228	logical or
?	ord	E393	or, horizontal dash
?	order	2134	order of (script small o)
?	oror	E50E	two logical or
?	orslope	E3AE	sloping large or
?	orv	E392	or with middle stem
	par	2225	parallel
?	parsl	E382	parallel, slanted
∂	part	2202	partial differential
?	permil	2030	per thousand
⊥	perp	22A5	perpendicular
?	pertenk	2031	per 10 thousand
?	phmmat	2133	physics M-matrix (script capital M)
?	pointint	E376	integral around a point operator
?	Prime	2033	double prime or second
'	prime	2032	prime or minute
?	profalar	232E	all-around profile
?	profline	2312	profile of a line
?	profsurf	2313	profile of a surface
∞	prop	221D	is proportional to
?	qint	E378	quadruple integral operator
?	qprime	E371	quadruple prime
?	quatint	E377	quaternion integral operator
√	radic	221A	radical
?	Rang	300B	right angle bracket, double
)	rang	3009	right angle bracket
⇒	rArr	21D2	implies
?	rbbrk	3015	right broken bracket
?	roang	3019	right open angular bracket
?	robrk	301B	right open bracket
?	ropar	E37A	right open parenthesis
?	rppolint	E397	line integration, rectangular path around pole
?	scpolint	E398	line integration, semi-circular path around pole
~	sim	223C	similar
?	simdot	E38B	similar, dot
≈	sime	2243	similar, equals
?	smeparsl	E385	similar, parallel, slanted, equal
■	square	25A1	square
?	squarf	25A0	square, filled
?	strns	E380	straightness
⊂	sub	2282	subset or is implied by

\subseteq	sube	2286	subset, equals
\supset	sup	2283	superset or implies
\supseteq	supe	2287	superset, equals
$\overset{?}{\cdot}$	tdot	20DB	three dots above
\therefore	there4	2234	therefore
$\int\!\!\int\!\!\int$	tint	222D	triple integral operator
\top	top	22A4	top
$\overset{?}{\top\!\!\bot}$	topbot	2336	top and bottom
$\overset{?}{\circ}$	topcir	E383	top, circle below
$\overset{?}{\prime}$	tprime	2034	triple prime
$\overset{?}{\cdot}$	utdot	22F0	three dots, ascending
$\overset{?}{\angle}$	uwangle	E3AB	large upward pointing angle
$\overset{?}{\angle}$	vangrt	22BE	right angle, variant (with arc)
$\overset{?}{\vee}$	veeeq	225A	logical or, equals
\parallel	Verbar	2016	double vertical bar
$\overset{?}{\wedge}$	wedgeq	2259	corresponds to (wedge, equals)
$\overset{?}{\boxplus}$	xnis	E3A3	large contains, vertical bar on horizontal stroke

<> *Negated Relations*

$\overset{?}{\gt}$	gnap	E411	greater, not approximate
$\overset{?}{\geq}$	gnE	2269	greater, not double equals
$\overset{?}{\neq}$	gne	2269	greater, not equals
$\overset{?}{\sim}$	gnsim	22E7	greater, not similar
$\overset{?}{\gt}$	gvnE	E2A1	gt, vert, not double equals
$\overset{?}{\lt}$	lnap	E2A2	less, not approximate
$\overset{?}{\geq}$	lnE	2268	less, not double equals
$\overset{?}{\neq}$	lne	2268	less, not equals
$\overset{?}{\sim}$	lnsim	22E6	less, not similar
$\overset{?}{\lt}$	lvnE	E2A4	less, vert, not double equals
$\overset{?}{\approx}$	nap	2249	not approximate
$\overset{?}{\approx}$	napE	E2C7	not approximately equal or equal to
$\overset{?}{\approx}$	napid	E2BC	not approximately identical to
$\overset{?}{\cong}$	ncong	2247	not congruent with
$\overset{?}{\cong}$	ncongdot	E2C5	not congruent, dot
$\overset{?}{\equiv}$	nequiv	2262	not identical with
$\overset{?}{\geq}$	ngE	2271	not greater, double equals
$\overset{?}{\geq}$	nge	E2A6	not greater-than-or-equal
$\overset{?}{\geq}$	nges	2271	not gt-or-equals, slanted

?	nGg	E2CE	not triple greater than
?	ngsim	2275	not greater, similar
?	nGt	E2CA	not, vert, much greater than
?	ngt	226F	not greater-than
?	nGtv	E2CC	not much greater than, variant
?	nLE	2270	not less, double equals
?	nle	E2A7	not less-than-or-equal
?	nles	2270	not less-or-equals, slant
?	nLl	E2CD	not triple less than
?	nlsim	2274	not less, similar
?	nLt	E2C9	not, vert, much less than
?	nlt	226E	not less-than
?	nlttri	22EA	not left triangle
?	nlttrie	22EC	not left triangle, equals
?	nLtv	E2CB	not much less than, variant
?	nmid	2224	negated mid
?	npar	2226	not parallel
?	npr	2280	not precedes
?	nprcue	22E0	not curly precedes, equals
?	npre	E412	not precedes, equals
?	nrtri	22EB	not right triangle
?	nrtrie	22ED	not right triangle, equals
?	nsc	2281	not succeeds
?	nsccue	22E1	not succeeds, curly equals
?	nsce	E413	not succeeds, equals
?	nsim	2241	not similar
?	nsime	2244	not similar, equals
?	nsmid	E2AA	negated short mid
?	nspar	E2AB	not short par
?	nsqsube	22E2	not, square subset, equals
?	nsqsupe	22E3	not, square superset, equals
?	nsu	2284	not subset
?	nsuE	2288	not subset, double equals
?	nsu	2288	not subset, equals
?	nsup	2285	not superset
?	nsuE	2289	not superset, double equals
?	nsupe	2289	not superset, equals
?	ntgl	2279	not greater, less
?	ntl	2278	not less, greater
?	ntvgl	2279	not, vert, greater, less

?	ntvlg	2278	not, vert, less, greater
?	nvap	E2C6	not, vert, approximate
?	nVDash	22AF	not double vert, double dash
?	nVdash	22AE	not double vertical, dash
?	nvDash	22AD	not vertical, double dash
?	nvdash	22AC	not vertical, dash
?	nvge	2271	not, vert, greater-than-or-equal
?	nvgt	226F	not, vert, greater-than
?	nvle	2270	not, vert, less-than-or-equal
?	nvlt	226E	not, vert, less-than
?	nvltrle	E2D0	not, vert, left triangle, equals
?	nvrtle	E2CF	not, vert, right triangle, equals
?	nvsim	E415	not, vert, similar
?	parsim	E2C8	parallel, similar
?	prnap	22E8	precedes, not approx
?	prnE	E2B3	precedes, not double equals
?	prnsim	22E8	precedes, not similar
?	rnmid	E2D1	reverse nmid
?	scnap	22E9	succeeds, not approx
?	scnE	E2B5	succeeds, not double equals
?	scnsim	22E9	succeeds, not similar
?	simne	2246	similar, not equals
?	solbar	E416	solidus, bar through
?	subnE	228A	subset, not double equals
?	subne	228A	subset, not equals
?	supnE	228B	superset, not double equals
?	supne	228B	superset, not equals
?	vnsup	2284	not subset, variant
?	vnsup	2285	not superset, variant
?	vsubnE	E2B8	subset not double equals, variant
?	vsubne	E2B9	subset, not equals, variant
?	vsupnE	E2BB	super not double equals, variant
?	vsupne	E2BA	superset, not equals, variant

<> Ordinary Symbols

?	apE	E315	approximately equal or equal to
?	ape	224A	approximate, equals
?	apid	224B	approximately identical to

\asymp	asym	224D	asymptotically equal to
$\overline{\overline{}}$	Barv	E311	vert, double bar (over)
\cong	bcong	224C	reverse congruent
\leadsto	bepsi	E420	such that
\bowtie	bowtie	22C8	bowtie
\sim	bsim	223D	reverse similar
\doteq	bsime	22CD	reverse similar, equals
\supseteq	bsolhsup	E34D	reverse solidus, subset
\bumpeq	bump	224E	bumpy equals
\Bumpeq	bumpe	224F	bumpy equals, equals
\circ	cire	2257	circle, equals
$::$	Colon	2237	two colons
$::=$	Colone	E30E	double colon, equals
\colon	colone	2254	colon, equals
$\dot{\cong}$	congdot	E314	congruent, dot
\subset	csub	E351	subset, closed
\subseteq	csube	E353	subset, closed, equals
\supset	csup	E352	superset, closed
\supseteq	csupe	E354	superset, closed, equals
\curlyeqpre	cuepr	22DE	curly equals, precedes
\curlyeqsucc	cuesc	22DF	curly equals, succeeds
\dashv	Dashv	E30F	double dash, vertical
\dashv	dashv	22A3	dash, vertical
$\stackrel{*}{=}$	easter	225B	equal, asterisk above
$\circ =$	ecir	2256	circle on equals sign
$=:$	ecolon	2255	equals, colon
\equiv	eDDot	E309	equal with four dots
\equiv	eDot	2251	equals, even dots
\equiv	efDot	2252	equals, falling dots
\geq	eg	E328	equal-or-greater
\gtrsim	egs	22DD	equal-or-gtr, slanted
\gtrdot	egsdot	E324	equal-or-greater, slanted, dot inside
\leq	el	E327	equal-or-less
\lesssim	els	22DC	eq-or-less, slanted
\lesdot	elsdot	E323	equal-or-less, slanted, dot inside
$\stackrel{?}{=}$	equest	225F	equal with questionmark
\equiv	equivDD	E318	equivalent, four dots above
\equiv	erDot	2253	equals, rising dots
$\stackrel{\cdot}{=}$	esdot	2250	equals, single dot above
\approx	Esim	E317	equal, similar

\boxminus	esim	2242	equals, similar
\boxplus	fork	22D4	pitchfork
\boxtimes	forkv	E31B	fork, variant
\frown	frown	2322	down curve
\gtrsim	gap	2273	greater, approximate
\gtrless	gE	2267	greater, double equals
\gtrless	gEl	22DB	gt, double equals, less
\gtrless	geI	22DB	greater, equals, less
\gtrless	ges	E421	gt-or-equal, slanted
\gtrless	gescc	E358	greater than, closed by curve, equal, slanted
\gtrless	gesdot	E31E	greater-than-or-equal, slanted, dot inside
\gtrless	gesdoto	E320	greater-than-or-equal, slanted, dot above
\gtrless	gesdotoI	E322	greater-than-or-equal, slanted, dot above left
\gtrless	gesI	E32C	greater, equal, slanted, less
\gtrless	gesles	E332	greater, equal, slanted, less, equal, slanted
\gtrless	Gg	22D9	triple gtr-than
\gtrless	gl	2277	greater, less
\gtrless	gla	E330	greater, less, apart
\gtrless	glE	E32E	greater, less, equal
\gtrless	glj	E32F	greater, less, overlapping
\gtrsim	gsim	2273	greater, similar
\gtrsim	gsime	E334	greater, similar, equal
\gtrsim	gsiml	E336	greater, similar, less
\gtrless	Gt	226B	double greater-than sign
\gtrless	gtcc	E356	greater than, closed by curve
\gtrless	gtcir	E326	greater than, circle inside
\gtrless	gtdot	22D7	greater than, with dot
\gtrless	gtquest	E32A	greater than, questionmark above
\gtrless	gtrarr	E35F	greater than, right arrow
\gtrless	homtht	223B	homothetic
\gtrless	lap	2272	less, approximate
\gtrless	lat	E33A	larger than
\gtrless	late	E33C	larger than or equal
\gtrless	lates	E33E	larger than or equal, slanted
\gtrless	lE	2266	less, double equals
\gtrless	lEg	22DA	less, double equals, greater
\gtrless	leg	22DA	less, equals, greater
\gtrless	les	E425	less-than-or-equals, slant
\gtrless	lescc	E357	less than, closed by curve, equal, slanted
\gtrless	lesdot	E31D	less-than-or-equal, slanted, dot inside

\lesdoto	E31F	less-than-or-equal, slanted, dot above
\lesdotor	E321	less-than-or-equal, slanted, dot above right
\lesg	E32B	less, equal, slanted, greater
\lesges	E331	less, equal, slanted, greater, equal, slanted
\lg	2276	less, greater
\lgE	E32D	less, greater, equal
Ll	22D8	triple less-than
\lsim	2272	less, similar
\lsime	E333	less, similar, equal
\lsimg	E335	less, similar, greater
Lt	226A	double less-than sign
\ltcc	E355	less than, closed by curve
\ltcir	E325	less than, circle inside
\ltdot	22D6	less than, with dot
\ltlarr	E35E	less than, left arrow
\ltquest	E329	less than, questionmark above
\ltrie	22B4	left triangle, equals
$mcomma$	E31A	minus, comma above
$mDDot$	223A	minus with four dots, geometric properties
\mid	2223	mid
\mlcp	E30A	transversal intersection
\models	22A7	models
\mstpos	223E	most positive
Pr	E35C	double precedes
\prec	227A	precedes
\prap	227E	precedes, approximate
\prcue	227C	precedes, curly equals
\prE	227C	precedes, double equals
\pre	227C	precedes, equals
\prsim	227E	precedes, similar
\prurel	22B0	element precedes under relation
$ratio$	2236	ratio
\rtrie	22B5	right triangle, equals
\rtriltri	E359	right triangle above left triangle
Sc	E35D	double succeeds
\succ	227B	succeeds
\scap	227F	succeeds, approximate
\sccue	227D	succeeds, curly equals
\scE	227E	succeeds, double equals
\succeq	227D	succeeds, equals

?	<code>scsim</code>	227F	succeeds, similar
?	<code>sdote</code>	E319	equal, dot below
?	<code>sfrown</code>	E426	small down curve
?	<code>simg</code>	E30C	similar, greater
?	<code>simgE</code>	E338	similar, greater, equal
?	<code>siml</code>	E30B	similar, less
?	<code>simlE</code>	E337	similar, less, equal
?	<code>smid</code>	E301	shortmid
?	<code>smile</code>	2323	up curve
?	<code>smt</code>	E339	smaller than
?	<code>smtE</code>	E33B	smaller than or equal
?	<code>smtes</code>	E33D	smaller than or equal, slanted
?	<code>spar</code>	E302	short parallel
?	<code>sqsub</code>	228F	square subset
?	<code>sqsube</code>	2291	square subset, equals
?	<code>sqsup</code>	2290	square superset
?	<code>sqsupe</code>	2292	square superset, equals
?	<code>ssmile</code>	E303	small up curve
?	<code>Sub</code>	22D0	double subset
?	<code>subE</code>	2286	subset, double equals
?	<code>subedot</code>	E34F	subset, equals, dot
?	<code>submult</code>	E343	subset, multiply
?	<code>subplus</code>	E341	subset, plus
?	<code>subrarr</code>	E33F	subset, right arrow
?	<code>subsim</code>	E345	subset, similar
?	<code>subsub</code>	E349	subset above subset
?	<code>subsup</code>	E347	subset above superset
?	<code>Sup</code>	22D1	double superset
?	<code>supdsub</code>	E34C	superset, subset, dash joining them
?	<code>supE</code>	2287	superset, double equals
?	<code>supedot</code>	E350	superset, equals, dot
?	<code>suphsol</code>	E34E	superset, solidus
?	<code>suphsub</code>	E34B	superset, subset
?	<code>suplarr</code>	E340	superset, left arrow
?	<code>supmult</code>	E344	superset, multiply
?	<code>supplus</code>	E342	superset, plus
?	<code>supsim</code>	E346	superset, similar
?	<code>supsub</code>	E348	superset above subset
?	<code>supsup</code>	E34A	superset above superset
?	<code>thkap</code>	E306	thick approximate

	thksim	E429	thick similar
	topfork	E31C	fork with top
	trie	225C	triangle, equals
	twixt	226C	between
	Vbar	E30D	double vert, bar (under)
	vBar	E310	vert, double bar (under)
	vBarv	E312	double bar, vert over and under
	VDash	22AB	double vert, double dash
	Vdash	22A9	double vertical, dash
	vDash	22A8	vertical, double dash
	vdash	22A2	vertical, dash
	Vdashl	E313	vertical, dash (long)
	vltri	22B2	left triangle, open, variant
	vprop	221D	proportional, variant
	vrtri	22B3	right triangle, open, variant
	Vvdash	22AA	triple vertical, dash

<> Ordinary

	ang	2220	angle
	ange	E2D6	angle, equal
	angmsd	2221	angle-measured
	angmsdaa	E2D9	angle-measured, arrow, up, right
	angmsdab	E2DA	angle-measured, arrow, up, left
	angmsdac	E2DB	angle-measured, arrow, down, right
	angmsdad	E2DC	angle-measured, arrow, down, left
	angmsdae	E2DD	angle-measured, arrow, right, up
	angmsdaf	E2DE	angle-measured, arrow, left, up
	angmsdag	E2DF	angle-measured, arrow, right, down
	angmsdah	E2E0	angle-measured, arrow, left, down
	angrtvb	E418	right angle-measured
	angrtvbd	E2E1	right angle-measured, dot
	bbrk	E2EE	bottom square bracket
	bbrktbrk	E419	bottom above top square bracket
	bemptyv	E41A	reversed circle, slash
	beth	2136	beth, Hebrew
	boxbox	E2E6	two joined squares
	bprime	2035	reverse prime
	bsemi	E2ED	reverse semi-colon

\emptyset	cemptyv	E2E8	circle, slash, small circle above
\circ	cirE	E41B	circle, two horizontal stroked to the right
\circ	cirscir	E41C	circle, small circle to the right
\complement	comp	2201	complement sign
\daleth	daleth	2138	daleth, Hebrew
\emptyset	demptyv	E2E7	circle, slash, bar above
ℓ	ell	2113	cursive small l
\emptyset	empty	E2D3	letter O slashed
\emptyset	emptyv	2205	circle, slash
\gimel	gimel	2137	gimel, Hebrew
ι	iiota	2129	inverted iota
\Im	image	2111	imaginary
\imath	imath	0131	small i, no dot
\emptyset	jmath	E2D4	small j, no dot
\emptyset	laemptyv	E2EA	circle, slash, left arrow above
\triangleleft	lltri	E2E5	lower left triangle
\triangleleft	lrtri	E2E3	lower right triangle
\mho	mho	2127	conductance
\nang	nang	E2D8	not, vert, angle
\nexists	nexist	2204	negated exists
\circ	oS	E41D	capital S in circle
\hbar	plank	E2D5	Planck's over 2pi
\hbar	plankv	210F	variant Planck's over 2pi
\emptyset	raemptyv	E2E9	circle, slash, right arrow above
\sphericalangle	range	E2D7	reverse angle, equal
\Re	real	211C	real
\top	tbrk	E2EF	top square bracket
\trapezium	trpezium	E2EC	trapezium
\triangleleft	ultri	E2E4	upper left triangle
\triangleleft	urtri	E2E2	upper right triangle
\zigzag	vzigzag	E2EB	vertical zig-zag line
\wp	weierp	2118	Weierstrass p

Synonym entities

If, after reading the previous chapter, you still think that entities form a well defined clean language, this chapter will destroy your dreams.

<> *Arrow relations*

	bkarow	rbarr
	circlearrowleft	olarr
	circlearrowright	orarr
	curvearrowleft	cularr
	curvearrowright	curarr
	dbkarow	rBarr
	DoubleDownArrow	dArr
	DoubleLeftRightArrow	hArr
	DoubleLongLeftArrow	xlArr
	DoubleLongLeftRightArrow	xhArr
	DoubleLongRightArrow	xrArr
	DoubleUpArrow	uArr
	DoubleUpDownArrow	vArr
	Downarrow	dArr
	DownArrowUpArrow	duarr
	downdownarrows	ddarr
	downharpoonleft	dharl
	downharpoonright	dharr
	DownLeftVector	lhard
	DownRightVector	rhard
	drbkarow	RBarr
	Equilibrium	rlhar
	hksearrow	searhk
	hkswarrow	swarhk
	hookleftarrow	larrhk
	hookrightarrow	rarrhk
	LeftArrowRightArrow	lrarr
	leftarrowtail	larrtl
	LeftDownVector	dharl
	leftharpoondown	lhard
	leftharpoonup	lharu
	leftleftarrows	llarr
	LeftRightArrow	harr
	Leftrightarrow	hArr






	leftarrow	harr
	leftarrows	larr
	leftrightharpoons	lrhar
	leftrightsquigarrow	harrw
	LeftUpVector	uharl
	LeftVector	lharu
	Lleftarrow	lAarr
	LongLeftArrow	xlarr
	Longleftarrow	xlArr
	longleftarrow	xlarr
	LongLeftRightArrow	xharr
	Longleftrightarrow	xhArr
	longleftrightarrow	xharr
	longmapsto	xmap
	LongRightArrow	xrarr
	Longrightarrow	xrArr
	longrightarrow	xrarr
	looparrowleft	larrlp
	looparrowright	rarrlp
	LowerLeftArrow	swarr
	LowerRightArrow	searr
	Lsh	lsh
	mapsto	map
	multimap	mumap
	nearrow	nearr
	nLeftarrow	nlArr
	nleftarrow	nlarr
	nLeftrightarrow	nhArr
	nleftrightarrow	nharr
	nrightarrow	nrArr
	nrightarrow	nrarr
	nwarrow	nwarr
	ReverseEquilibrium	lrhar
	ReverseUpEquilibrium	duhar
	RightArrowLeftArrow	rlarr
	rightarrowtail	rarrtl
	RightDownVector	dharr
	rightharpoondown	rhard
	rightharpoonup	rharu
	rightleftarrows	rlarr







?	rightleftharpoons	rlhar
?	rightrightarrow	rrarr
?	rightsquigarrow	rarrw
?	RightTeeArrow	map
?	RightUpVector	uharr
?	RightVector	rharu
?	Rrightarrow	rAarr
?	Rsh	rsh
?	searrow	searr
?	ShortLeftArrow	slarr
?	ShortRightArrow	srarr
?	swarrow	swarr
?	toea	nesear
?	tosa	seswar
?	twoheadleftarrow	Larr
?	twoheadrightarrow	Rarr
?	Uparrow	uArr
?	UpArrowDownArrow	udarr
?	UpDownArrow	varr
?	Updownarrow	vArr
?	updownarrow	varr
?	UpEquilibrium	udhar
?	upharpoonleft	uharl
?	upharpoonright	uharr
?	UpperLeftArrow	nwarr
?	UpperRightArrow	nearr
?	upuparrows	uuarr

<> *Binary operators*











?	ast	midast
?	Backslash	setmn
?	barwedge	barwed
?	bigcap	xcap
?	bigcirc	xcirc
?	bigcup	xcup
?	bigodot	xodot
?	bigoplus	xoplus
?	bigotimes	xotime

Synonym entities
















	bigscup	xscup
	bigtriangledown	xdtri
	bigtriangleup	xutri
	biguplus	xuplus
	bigvee	xvee
	bigwedge	xwedge
	boxminus	minusb
	boxplus	plusb
	boxtimes	timesb
	cdot	sdot
	circledast	oast
	circledcirc	ocir
	circleddash	odash
	CircleDot	odot
	CircleMinus	ominus
	CirclePlus	oplus
	CircleTimes	otimes
	Coproduct	coprod
	curlyvee	cuvee
	curlywedge	cuwed
	ddagger	Dagger
	diamond	diam
	divideontimes	divonx
	dotminus	minusd
	dotplus	plusdo
	dotsquare	sdotb
	doublebarwedge	Barwed
	intercal	intcal
	Intersection	xcap
	intprod	iproduct
	leftthreetimes	lthree
	oslash	osol
	rightthreetimes	rthree
	setminus	setmn
	smallsetminus	ssetmn
	SquareIntersection	sqcap
	SquareUnion	sqcup
	Star	sstarf
	star	sstarf
	Sum	sum

	Union	xcup
	UnionPlus	uplus
	Vee	xvee
	VerticalTilde	wreath
	Wedge	xwedge
	wr	wreath

<> *Delimiters*

	LeftCeiling	lceil
	LeftFloor	lfloor
	llcorner	dlcorn
	lmoustache	lmoust
	lrcorner	drcorn
	RightCeiling	rceil
	RightFloor	rfloor
	rmoustache	rmoust
	ulcorner	ulcorn
	urcorner	urcorn

<> *General Technical*

\approx	approx	ap
	Because	becaus
	because	becaus
	blacksquare	squarf
	bot	bottom
◦	circ	compfn
	ClockwiseContourIntegral	cwconint
	Congruent	equiv
	ContourIntegral	conint
	CounterClockwiseContourIntegral	awconint
	Del	nabla
	DoubleContourIntegral	Conint
	DoubleLeftArrow	lArr
	DoubleRightArrow	rArr
	DoubleVerticalBar	par
	DownTee	top
	Element	isinv

\exists	Exists	exist
\forall	ForAll	forall
\geq	geq	ge
\geq	GreaterEqual	ge
\mathbb{I}	iiiint	qint
\mathbb{I}	iiint	tint
\Rightarrow	Implies	rArr
\in	in	isin
\int	Integral	int
\langle	langle	lang
\langle	LeftAngleBracket	lang
\leftarrow	Leftarrow	lArr
\leq	leq	le
$-$	MinusPlus	mnplus
\cdot	mp	mnplus
\notin	NotElement	notin
\neq	NotEqual	ne
\nrightarrow	NotReverseElement	notniva
\oint	oint	conint
\parallel	parallel	par
∂	PartialD	part
\propto	Proportional	prop
\propto	propto	prop
\rangle	rangle	rang
\rightrightarrows	ReverseElement	niv
\rightangle	RightAngleBracket	rang
\rightarrow	Rightarrow	rArr
\simeq	simeq	sime
\smallcirc	SmallCircle	compfn
$\sqrt{}$	Sqrt	radic
\subset	Subset	sub
\subset	subset	sub
\subseteq	subseteq	sube
\subseteq	SubsetEqual	sube
such that	SuchThat	ni
\supset	Superset	sup
\supseteq	SupersetEqual	supe
\supseteq	supset	sup
\supseteq	supseteq	supe
\therefore	Therefore	there4

?	therefore	there4
?	Tilde	sim
?	TildeEqual	sime
?	TildeFullEqual	cong
?	TildeTilde	ap
?	TripleDot	tdot
?	UpTee	perp
?	vee	or
?	Vert	Verbar
?	wedge	and

<> *Negated Relations*

?	gnapprox	gnap
?	gneq	gne
?	gneqq	gnE
?	gvertneqq	gvnE
?	lnapprox	lnap
?	lneq	lne
?	lneqq	lnE
?	lvertneqq	lvnE
?	napprox	nap
?	ngeq	nge
?	ngeqq	ngE
?	ngeqslant	nges
?	ngtr	ngt
?	nleq	nle
?	nleqq	nLE
?	nleqslant	nles
?	nless	nlt
?	NotCongruent	nequiv
?	NotDoubleVerticalBar	npar
?	NotGreater	ngt
?	NotGreaterEqual	nge
?	NotGreaterFullEqual	ngE
?	NotGreaterGreater	nGtv
?	NotGreaterLess	ntvgl
?	NotGreaterSlantEqual	nges
?	NotGreaterTilde	ngsim

?	NotLeftTriangle	nltri
?	NotLeftTriangleEqual	nltrie
?	NotLess	nl<
?	NotLessEqual	nle
?	NotLessFullEqual	nle
?	NotLessGreater	ntvlg
?	NotLessLess	nLtv
?	NotLessSlantEqual	nles
?	NotLessTilde	nlsim
?	NotPrecedes	npr
?	NotPrecedesEqual	npre
?	NotPrecedesSlantEqual	nprcue
?	NotRightTriangle	nrtri
?	NotRightTriangleEqual	nrtrie
?	NotSquareSubsetEqual	nsqsube
?	NotSquareSupersetEqual	nsqsupe
?	NotSubset	vnsup
?	NotSucceeds	nsc
?	NotSucceedsEqual	nsce
?	NotSucceedsSlantEqual	nsccue
?	NotSuperset	vnsup
?	NotTilde	nsim
?	NotTildeEqual	nsime
?	NotTildeFullEqual	ncong
?	NotTildeTilde	nap
?	NotVerticalBar	nmid
?	nparallel	npar
?	nprec	npr
?	npreceq	npre
?	nshortmid	nsmid
?	nshortparallel	nspar
?	nsimeq	nsime
?	nsupset	vnsup
?	nsupseteq	nsupe
?	nsupseteqq	nsupE
?	nsucc	nsc
?	nsucceq	nsce
?	nsupset	vnsup
?	nsupseteq	nsupe
?	nsupseteqq	nsupE

$\boxed{?}$	ntriangleleft	nltri
$\boxed{?}$	ntrianglelefteq	nltrie
$\boxed{?}$	nrtriangleright	nrtri
$\boxed{?}$	nrtrianglerighteq	nrtrie
$\boxed{?}$	precnapprox	prnap
$\boxed{?}$	precneqq	prnE
$\boxed{?}$	precnsim	prnsim
$\boxed{?}$	subsetneq	subne
$\boxed{?}$	subsetneqq	subnE
$\boxed{?}$	succnapprox	scnap
$\boxed{?}$	succneqq	scnE
$\boxed{?}$	succnsim	scnsim
$\boxed{?}$	supsetneq	supne
$\boxed{?}$	supsetneqq	supnE
$\boxed{?}$	varsubsetneq	vsubne
$\boxed{?}$	varsubsetneqq	vsubnE
$\boxed{?}$	varsupsetneq	vsupne
$\boxed{?}$	varsupsetneqq	vsupnE

<> Ordinary Symbols

$\boxed{?}$	approxeq	ape
$\boxed{?}$	backcong	bcong
$\boxed{?}$	backepsilon	bepsi
$\boxed{?}$	backsim	bsim
$\boxed{?}$	backsimeq	bsime
$\boxed{?}$	between	twixt
$\boxed{?}$	Bumpeq	bump
$\boxed{?}$	bumpeq	bumpe
$\boxed{?}$	circeq	cire
$\boxed{?}$	coloneq	colone
$\boxed{?}$	Cup	smile
$\boxed{?}$	CupCap	asympt
$\boxed{?}$	curlyeqprec	cuepr
$\boxed{?}$	curlyeqsucc	cuesc
$\boxed{?}$	ddotseq	eDDot
$\boxed{?}$	doteq	esdot
$\boxed{?}$	doteqdot	eDot
$\boxed{?}$	DotEqual	esdot

Synonym entities

?	DoubleRightTee	vDash
?	eqcirc	ecir
?	eqcolon	ecolon
?	eqsim	esim
?	eqslantgtr	egs
?	eqslantless	els
?	EqualTilde	esim
?	fallingdotseq	efDot
?	geqq	gE
?	geqslant	ges
?	gg	Gt
?	ggg	Gg
?	GreaterEqualLess	geI
?	GreaterFullEqual	gE
?	GreaterLess	gI
?	GreaterSlantEqual	ges
?	GreaterTilde	gsim
?	gtrapprox	gap
?	gtrdot	gtdot
?	gtreqless	geI
?	gtreqqless	gEl
?	gtrless	gI
?	gtrsim	gsim
?	HumpDownHump	bump
?	HumpEqual	bumpe
?	LeftTee	dashv
?	LeftTriangle	vltri
?	LeftTriangleEqual	ltrie
?	leqq	lE
?	leqslant	les
?	lessapprox	lap
?	lessdot	ltdot
?	lesseqgtr	leg
?	lesseqqgtr	lEg
?	LessEqualGreater	leg
?	LessFullEqual	lE
?	LessGreater	lg
?	lessgtr	lg
?	lesssim	lsim
?	LessSlantEqual	les

?	LessTilde	lsim
?	ll	Lt
?	NestedGreaterGreater	Gt
?	NestedLessLess	Lt
?	pitchfork	fork
?	prec	pr
?	precapprox	prap
?	preccurlyeq	prcue
?	Precedes	pr
?	PrecedesEqual	pre
?	PrecedesSlantEqual	prcue
?	PrecedesTilde	prsim
?	preceq	pre
?	precsim	prsim
?	Proportion	Colon
?	questeq	equest
?	RightTee	vdash
?	RightTriangle	vrtri
?	RightTriangleEqual	rtrie
?	risingdotseq	erDot
?	shortmid	smid
?	shortparallel	spar
?	smallfrown	sfrown
?	smallsmile	ssmile
?	sqsubset	sqsub
?	sqsubseteq	sqsube
?	sqsupset	sqsup
?	sqsupseteq	sqsupe
?	SquareSubset	sqsub
?	SquareSubsetEqual	sqsube
?	SquareSuperset	sqsup
?	SquareSupersetEqual	sqsupe
?	Subset	Sub
?	subseteqq	subE
?	succ	sc
?	succapprox	scap
?	succcurlyeq	sccue
?	Succeeds	sc
?	SucceedsEqual	sce
?	SucceedsSlantEqual	sccue

\square	SucceedsTilde	scsim
\square	succeq	sce
\square	succsim	scsim
\square	Supset	Sup
\square	supseteqq	supE
\square	thickapprox	thkap
\square	thicksim	thksim
\square	trianglelefteq	ltrie
\square	triangleq	trie
\square	trianglerighteq	rtrie
\square	varpropto	vprop
\square	vartriangleleft	vltri
\square	vartriangleright	vrtri
\square	VerticalBar	mid

<> Ordinary

\square	angle	ang
\square	backprime	bprime
\square	circledS	oS
\square	complement	comp
\emptyset	emptyset	empty
\square	hbar	plank
\square	hslash	plankv
\square	Im	image
\square	measuredangle	angmsd
\square	nexists	nexist
\square	NotExists	nexist
\square	Re	real
\square	varnothing	emptyv
\square	wp	weierp

Further reading

- <> *The MathML spec* You can fetch the latest version of this document, which is written by the MATHML committee, can be fetched from the www.w3c.org web-site. Depending on the state of development, you can grab the draft, recommendation or standard.
- <> *The T_EXbook* Once you have read this book, you will see why MATHML is not embraced by those who love to optimize the look and feel of their math formulas. Although your documents will be more consistent when you code in (content) MATHML, you also loose many fine points of math typesetting. Of course you can always fall back on the *annotation* element. This book is written by Donald Knuth and published by Addison Wesley.
- <> *The XML Companion* Written by Neil Bradley and published by Addison Wesley, this book is a good introduction to XML and its relatives. More compact but not less useful, is Robert Eckstein's *XML Pocket Reference*, published by O'Reilly.
- <> *XML in ConT_EXt* This document describes how you can use ConT_EXt for processing your XML documents. You may also want to take a look at the beginners manual, the reference manual, guides and examples that can be fetched from www.pragma-ade.com.
- <> *MathML in ConT_EXt* We have keyed in a lot of realistic MATHML examples and turned them into a document suitable for viewing on your computer display. Over time, this collection will grow.

